# Fundamentals of Atmospheric Modelling

# Part 1
# (modelling components and types)

**Todd Lane**

*ARC Centre of Excellence for Climate Extremes,*
*School of Earth Sciences, The University of Melbourne*

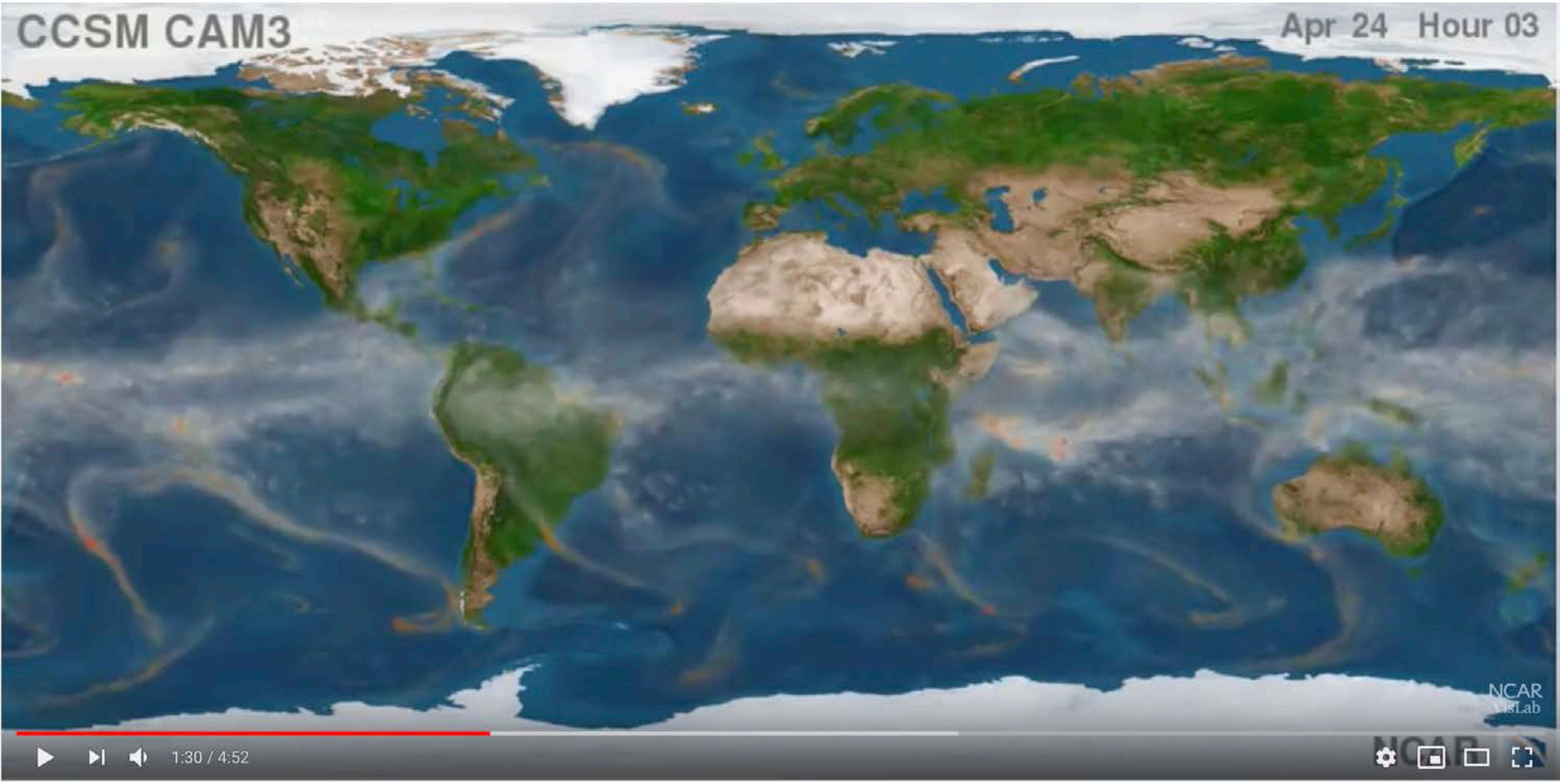
CLEX Winter School
24-28 June 2019

# Outline

- Types of atmospheric models
- Model components
- Numerical representation
- Sources of error
- Boundary conditions

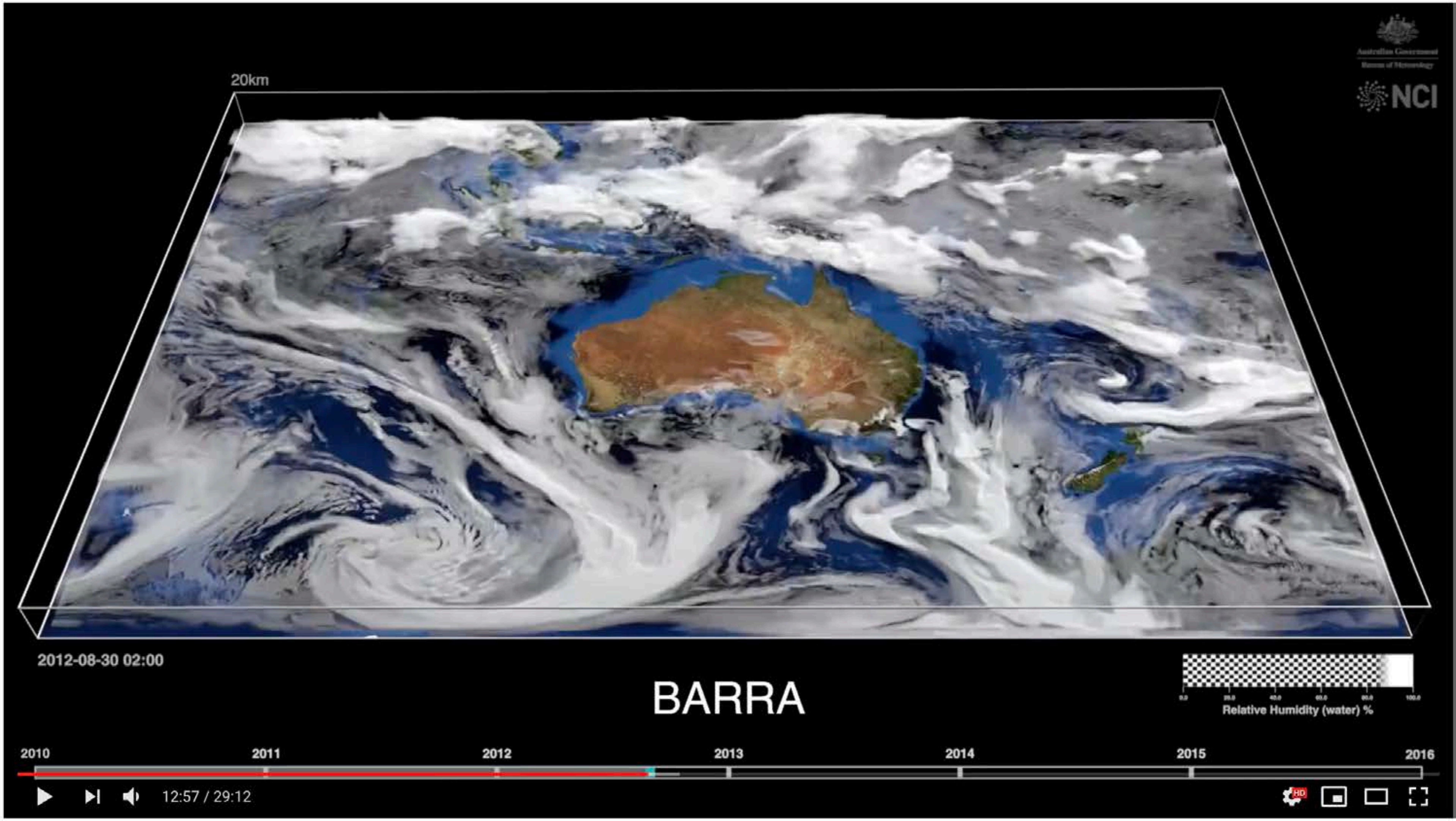
# Global model

NCAR climate model (~0.35 degree resolution)




https://www.youtube.com/watch?v=n0mupl4FZsQ

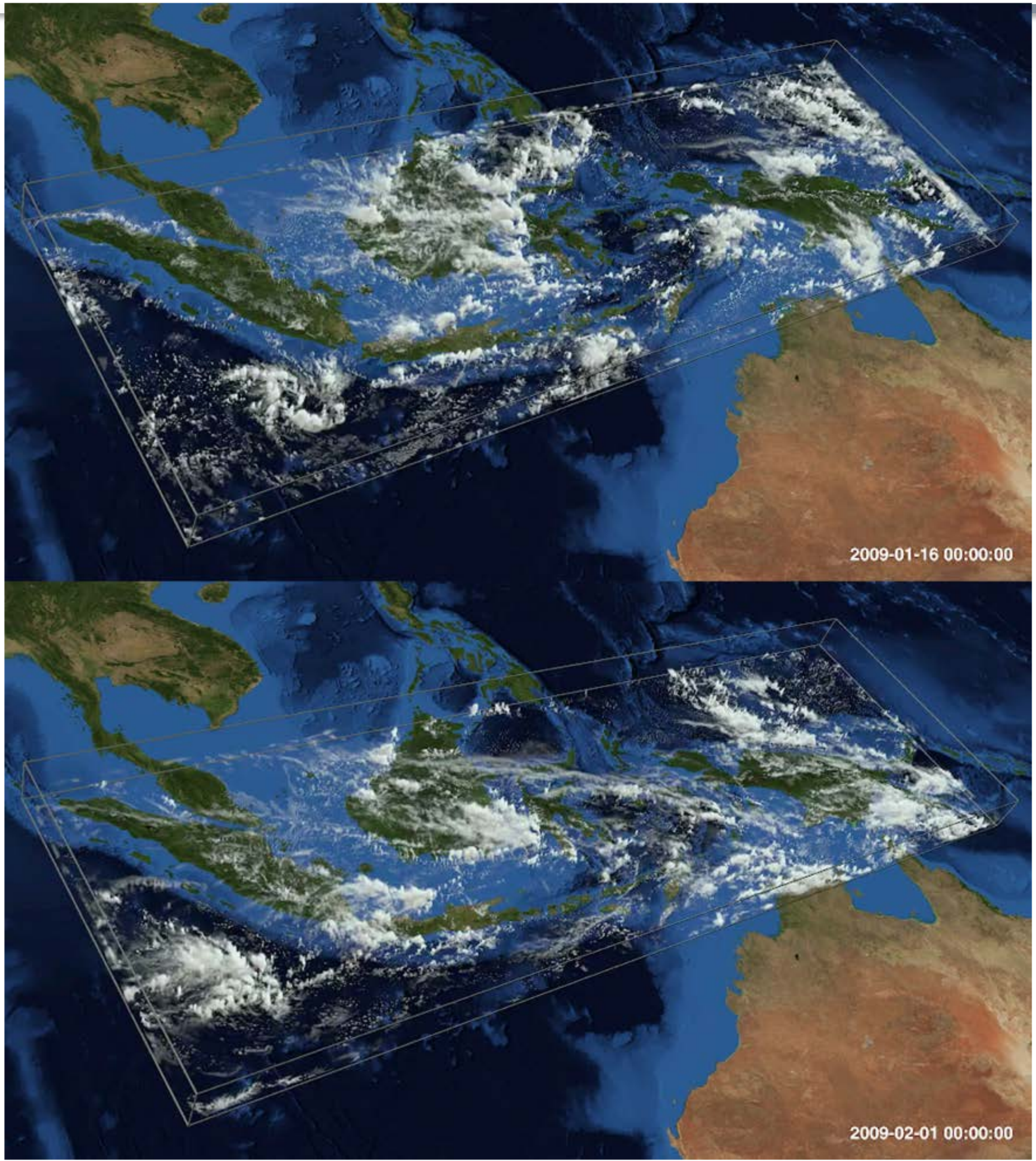
# Operational Numerical Weather Prediction Model




https://www.youtube.com/watch?time_continue=25&v=bBLsn_WCg8g

ACCESS-R (12 km)

# Regional Model

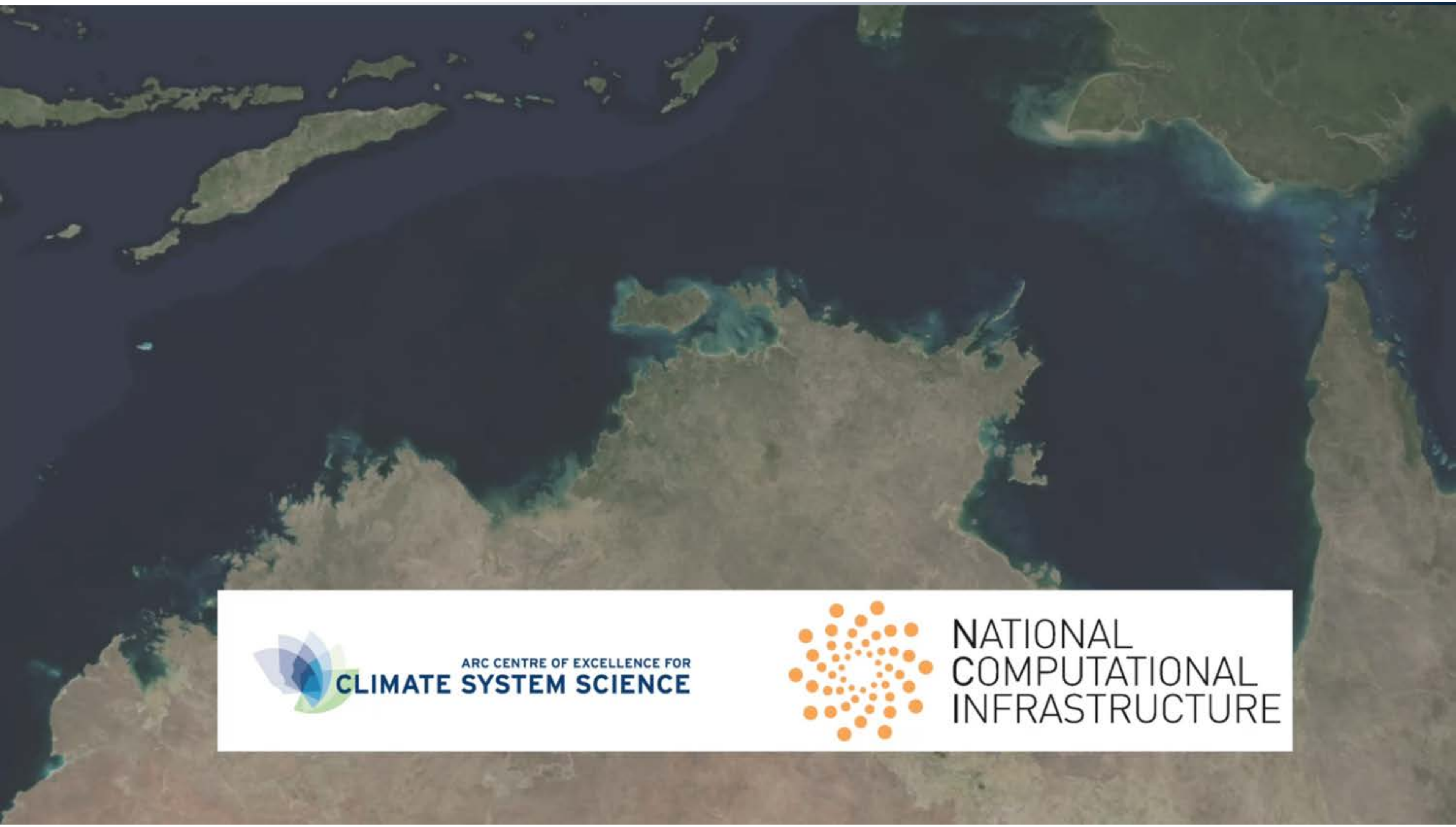



From Vincent & Lane (J Clim 2017)

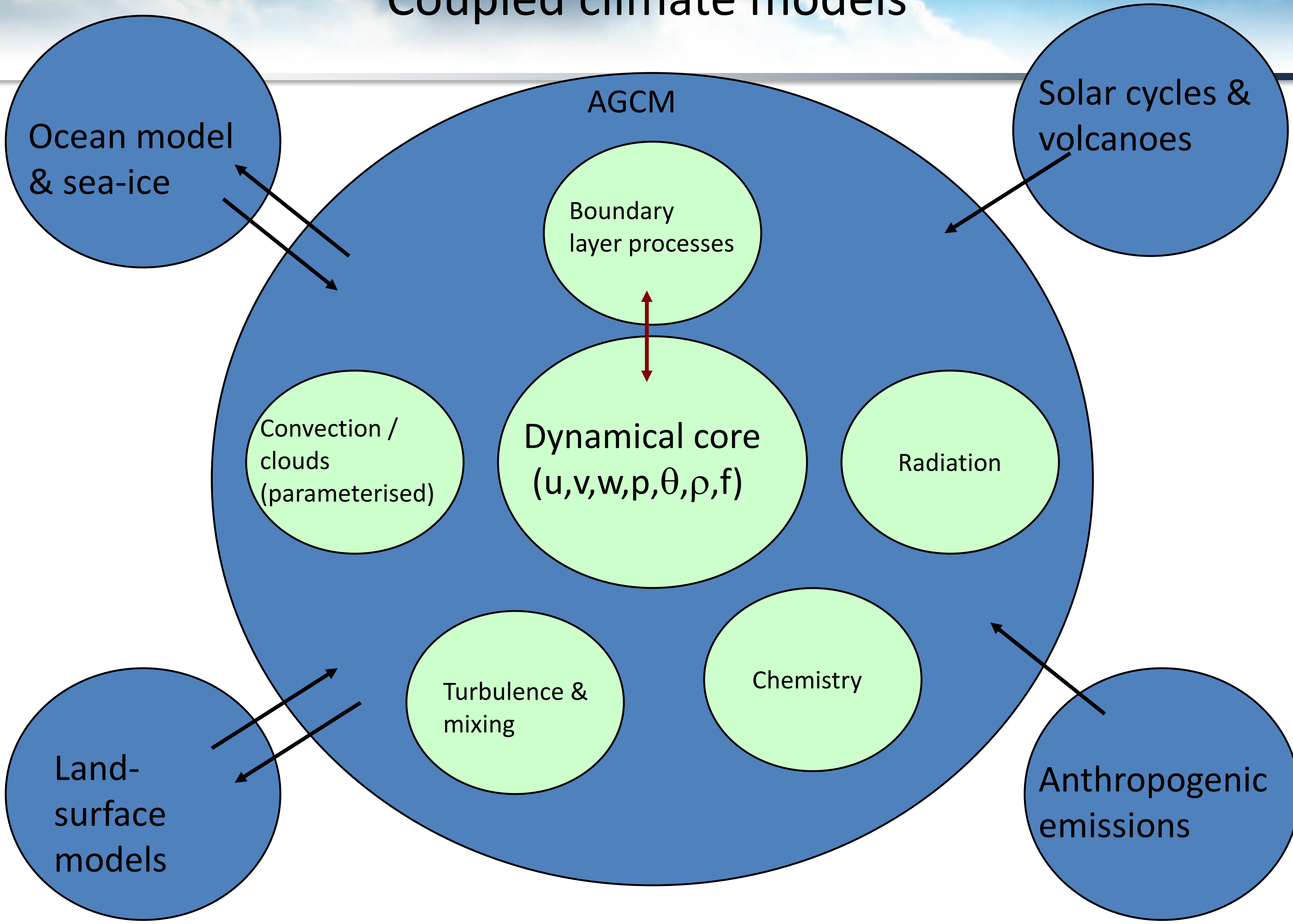
dx=4 km WRF simulations

Visualisation courtesy:
Drew Whitehouse NCI

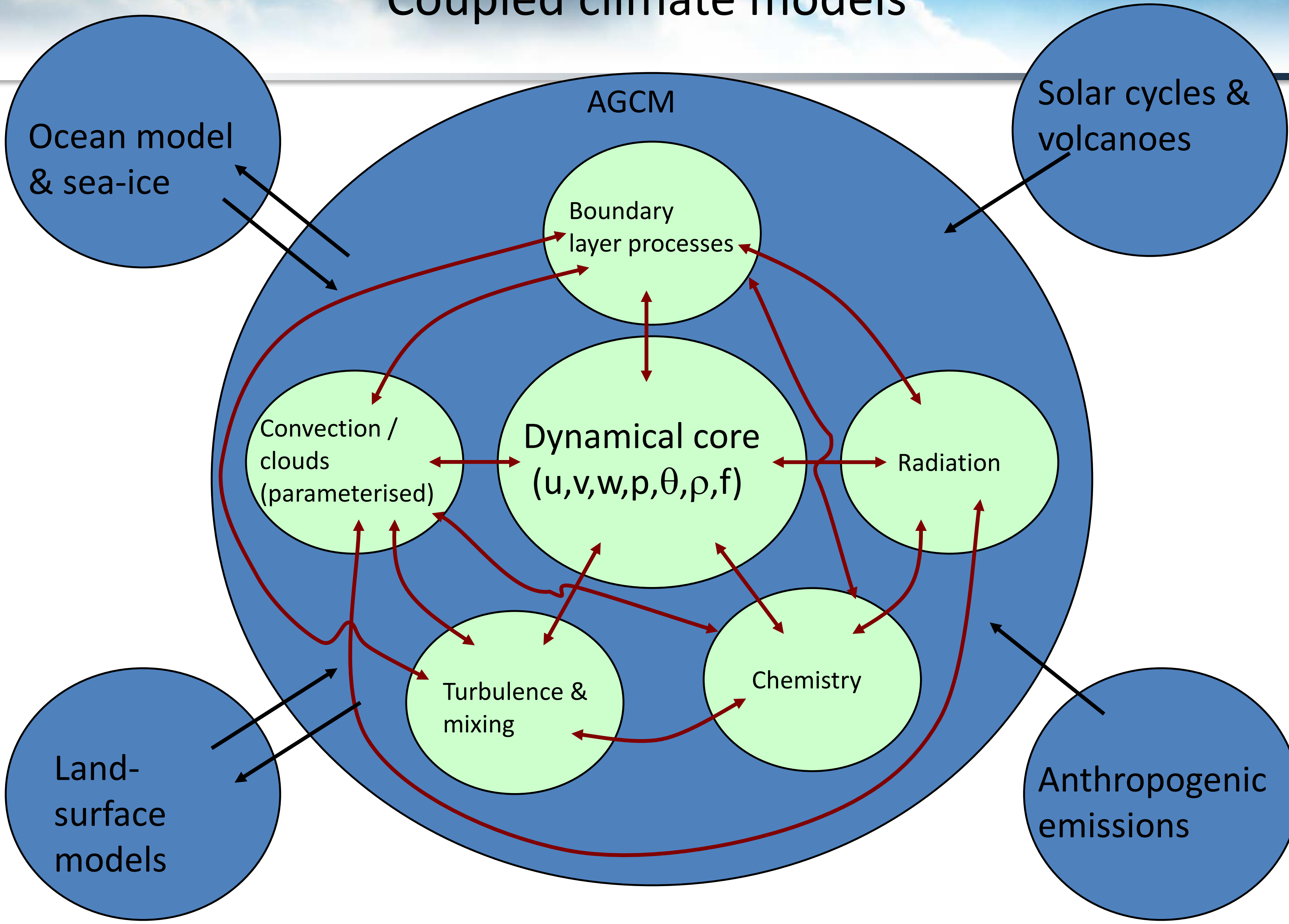
# Regional Model ( 0.4 km – cloud-resolving)



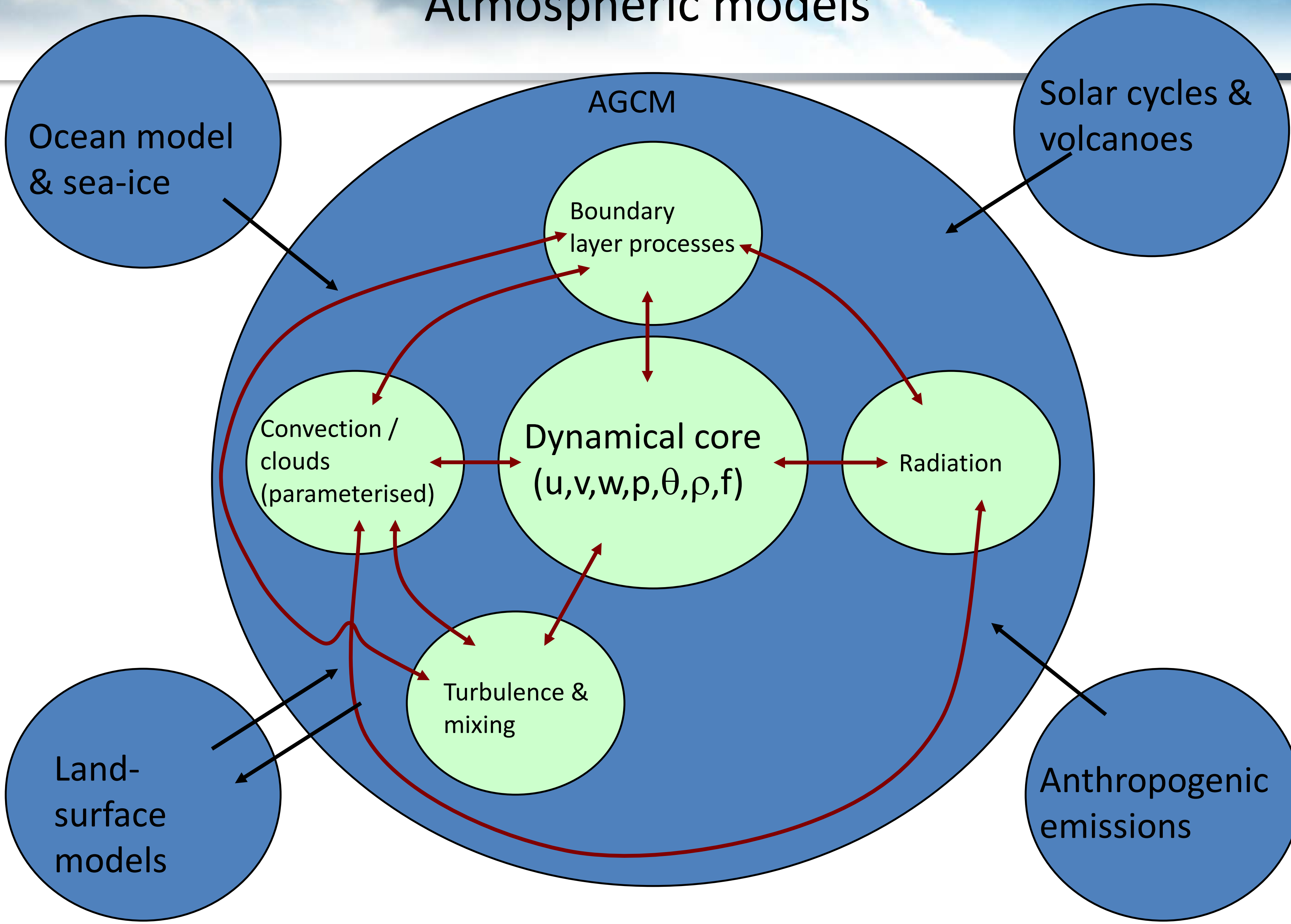
# Coupled climate models



# Coupled climate models



# Atmospheric models




Complexity varies by application and scale

# Dynamical core – forced equations of motion

Complete equations of motion:

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z$$

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \bullet (\rho \underline{U}) = 0$$

$$\frac{D\theta}{Dt} = 0 + F_\theta$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\theta = T\left[\frac{1000}{p}\right]^{R/Cp}$$

$$\underline{U} = (u, v, w)$$

θ is potential temperature
ρ is density
f  is coriolis

Exact form depends on assumptions / approximation and coordinate system

**F* is an external forcing from physics or parameterisations.**

# How are equations solved?

- Finite difference methods
- A combination of spectral / grid methods
  - Advantage – exact representation of horizontal derivatives on sphere
  - Physics step completed on grid
- Approximations to the equation set
  - (e.g., hydrostatic, anelastic, reduced compressibility, etc.)
- Specific vertical coordinate (e.g., height, pressure, terrain-following, etc.)

# Spectral methods (from ECMWF)

https://confluence.ecmwf.int/display/FCST/Spectral+representation+of+the+IFS

## Spectral representation of the IFS

Created by Paul Dando on Jul 17, 2015

The IFS uses a spectral transform method to solve numerically the equations governing the spatial and temporal evolution of the atmosphere. The idea is to fit a discrete representation of a field on a grid by a continuous function. This is achieved by expressing the function as a truncated series of spherical harmonics:

$$A(\lambda,\mu,\eta,t) = \sum_{l=0}^{T}\sum_{|m|\leq l}^{T} \psi_{lm}(\eta,t) Y_{lm}(\lambda,\mu) = \sum_{l=0}^{T}\sum_{|m|\leq l}^{T} \psi_{lm}(\eta,t) P_l^m(\mu) e^{im\lambda}$$

where $\mu = \sin\theta$ with $\lambda$ the longitude and $\theta$ the latitude of the grid point, T is the spectral truncation number and $Y_{lm}$ are the spherical harmonic functions which are products of the associated Legendre polynomials, $P_l^m(\mu)$ and the Fourier functions, $e^{im\lambda}$. The spectral coefficients $\psi_{lm}$ are computed from the discrete values known at each point of a Gaussian grid on the sphere by

- a Fast Fourier Transform in the zonal direction followed by
- a slow/fast Legendre transform in the meridional direction.

At each time step in the IFS:

- derivatives, semi-implicit correction and horizontal diffusion are computed in spectral space;
- explicit dynamics, semi-Lagrangian advection and physical parametrizations are computed in grid point space.

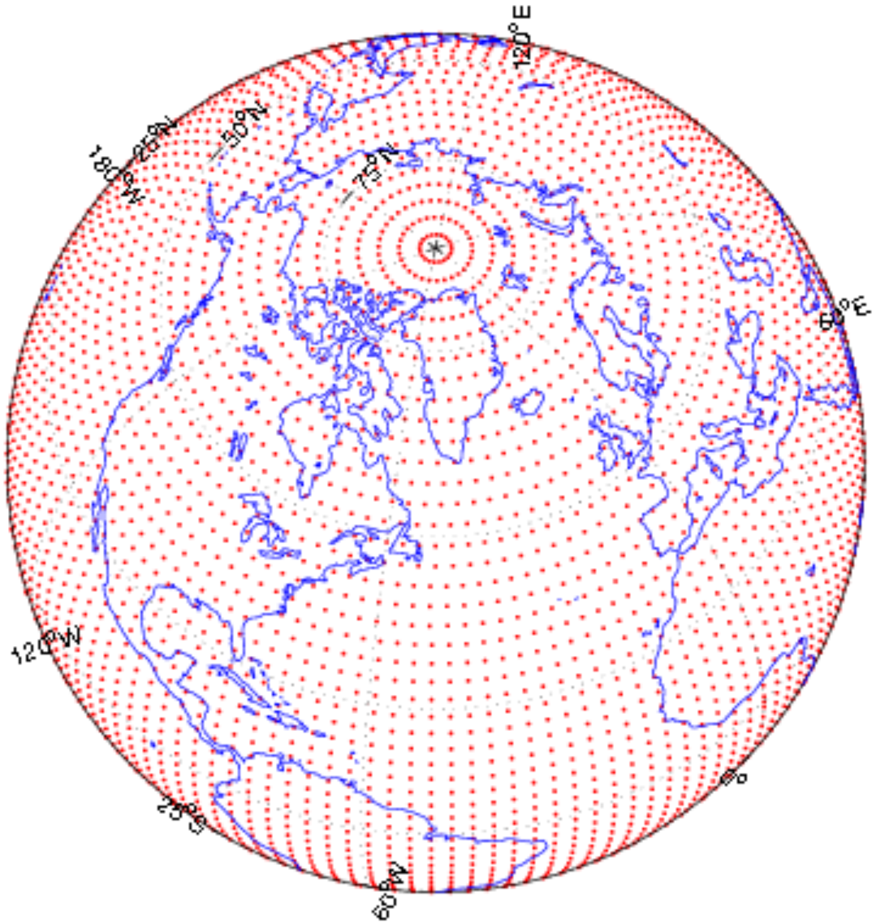
The representation in grid point space is on the **Gaussian grid**. The grid point resolution is determined by the spectral truncation number, T.

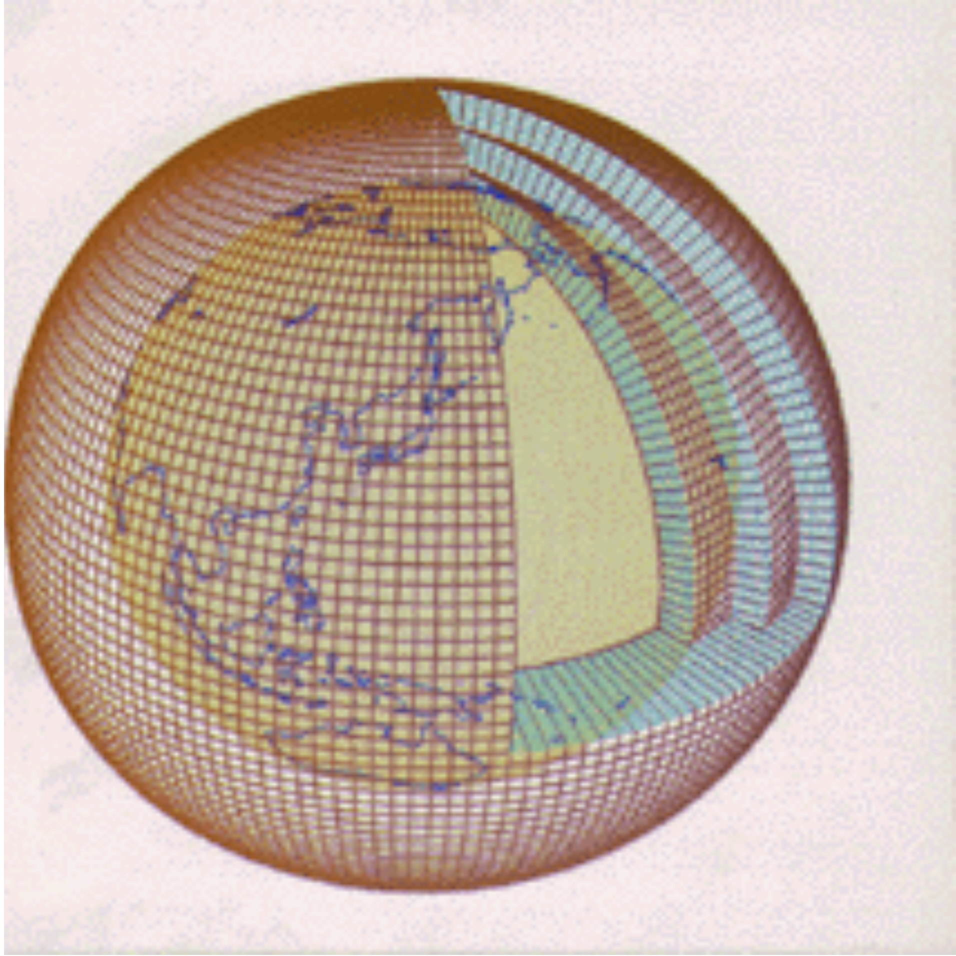
## Common Model Spectral Resolutions

| Truncation | lat x lon | km at Eq | deg at Eq |
|---|---|---|---|
| T21 | 32x64 | 625 | 5.61 |
| T42 | 64x128 | 310 | 2.79 |
| T62 | 94x192 | 210 | 1.89 |
| T63 | 96x192 | 210 | 1.88 |
| T85 | 128x256 | 155 | 1.39 |
| T106 | 160x320 | 125 | 1.12 |
| T159 | 240x480 | 83 | 0.75 |
| T255 | 256x512 | 60 | 0.54 |
| T382 | 576x1152 | 38 | 0.34 |
| T799 | 800x1600 | 25 | 0.22 |

https://climatedataguide.ucar.edu/climate-model-evaluation/common-spectral-model-grid-resolutions

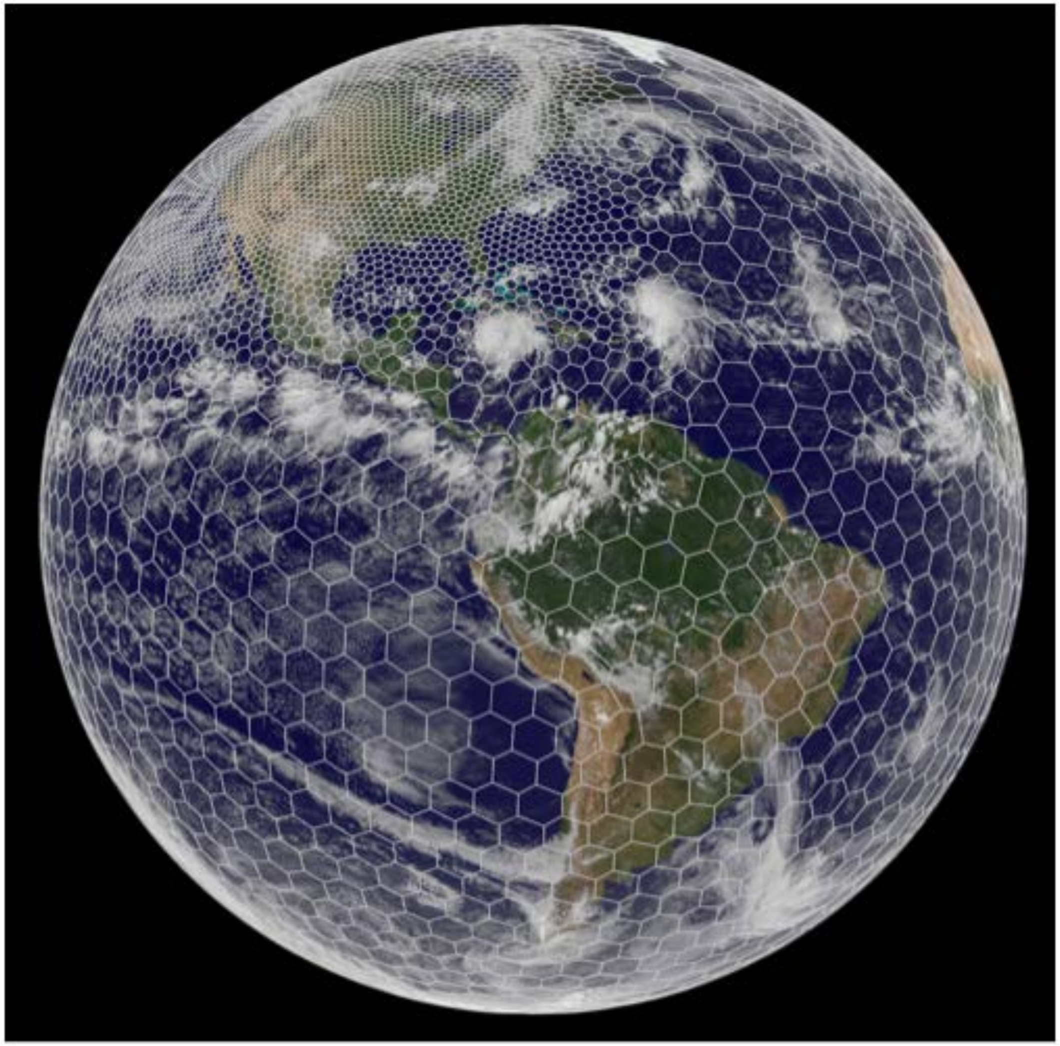
# Example global model (lat / lon-based) grid.



Challenges at poles as grids become poorly defined / too closely spaced (EC reduces the number of grid points near pole)

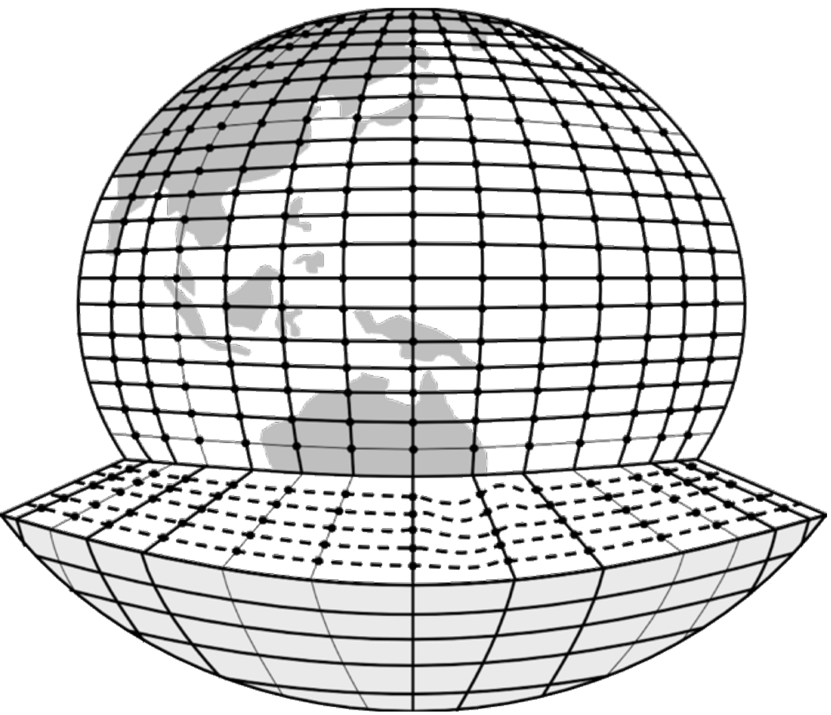
# Unstructured hexagonal global mesh



MPAS model

Variable resolution – alternate to ‘nesting’
Requires scale aware physics schemes
Avoids challenges at poles

# Computational limits



How many calculations does an atmospheric model alone* have to perform?

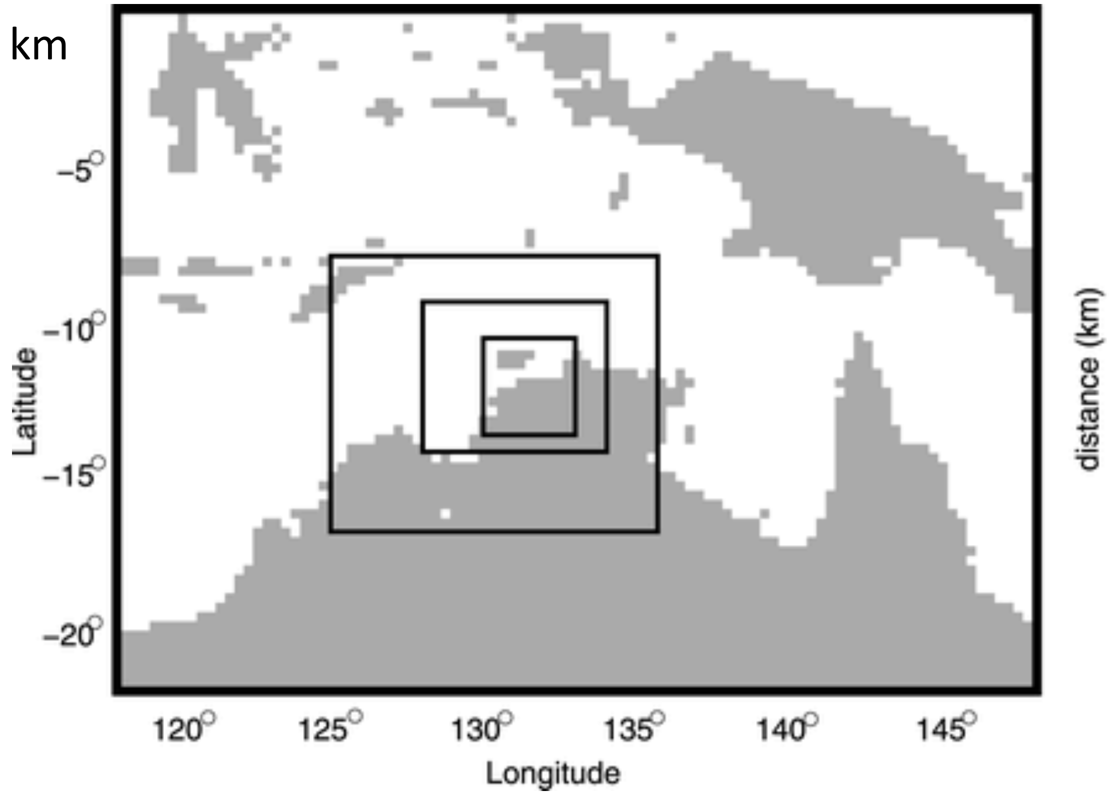
- 2.5 x 2.5$^{o}$ = about 10,000 cells
- 30 vertical layers = about 300,000 grid boxes
- At least 7 unknowns = about 2.1 million variables
- Assume 20 calculations (low estimate) for each variable = about 42 million calculations per time-step
- Time step of 30 minutes = about 2 billion calculations per day
- 100 year simulation = 73 trillion calculations
- Reducing grid spacing by half (in each dimension) can increase computational cost by a factor of 16.

*no chemistry, prognostic aerosol, or upper atmosphere

# Nested model simulations (dynamical downscaling / regional NWP):

ΔX = 34, 11.33, 3.78, 1.23 km

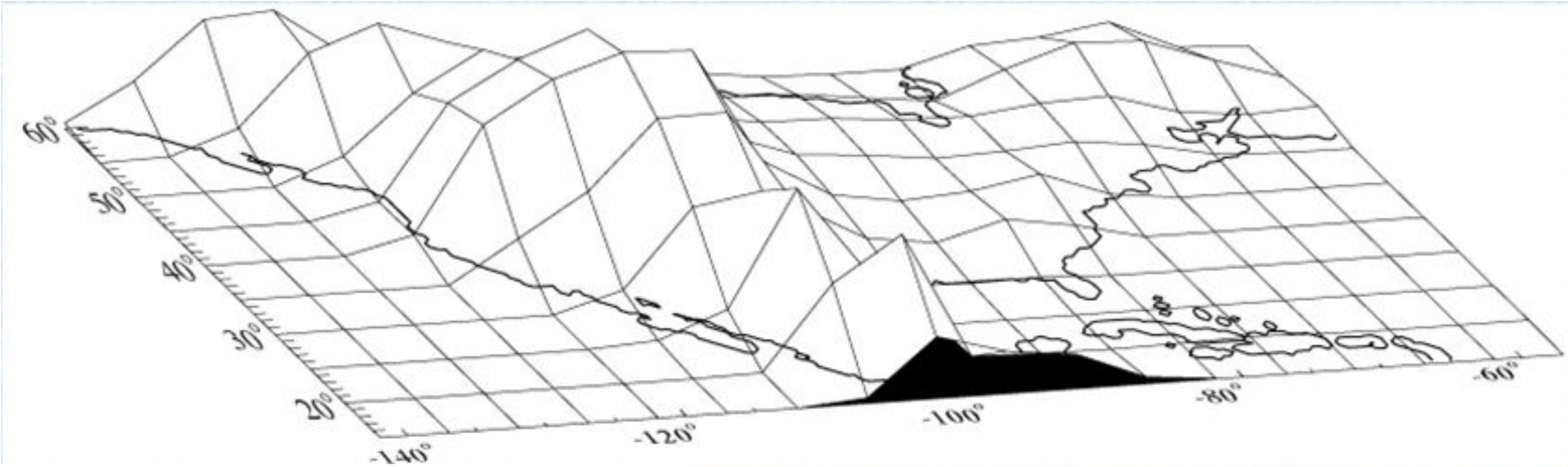
- Global model or analysis provides initial and boundary conditions to outer domain
    - Outer domain can be 'nudged' to global model/analysis
- Subsequent domains take their initial and boundary conditions from the next coarsest domains (one-way nesting)
- Depending on how model is coded there can be restrictive rules about the ratio of one grid resolution to the next
    - Most models aren't capable of vertical resolution changes from domain to domain
- Two-way nesting – high-resolution domains feed back on coarse-resolution domains. (Makes resolution sensitivity studies challenging)



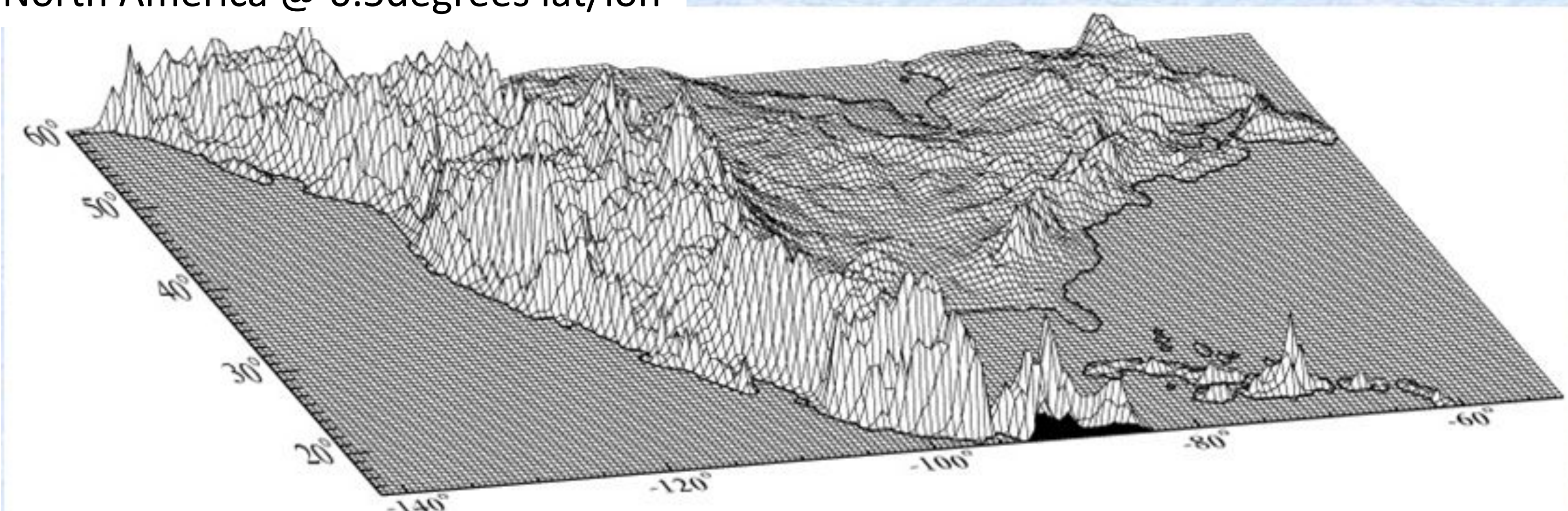

Wapler et al. (2010, *Monthly Weather Review*)

# Why increase resolution or use nesting (dynamical downscaling)?

North America @ 5degrees lat/lon



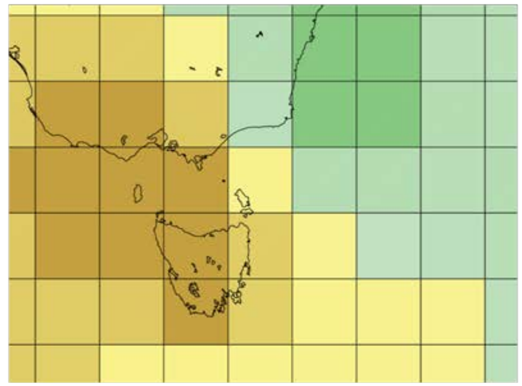

North America @ 0.5degrees lat/lon

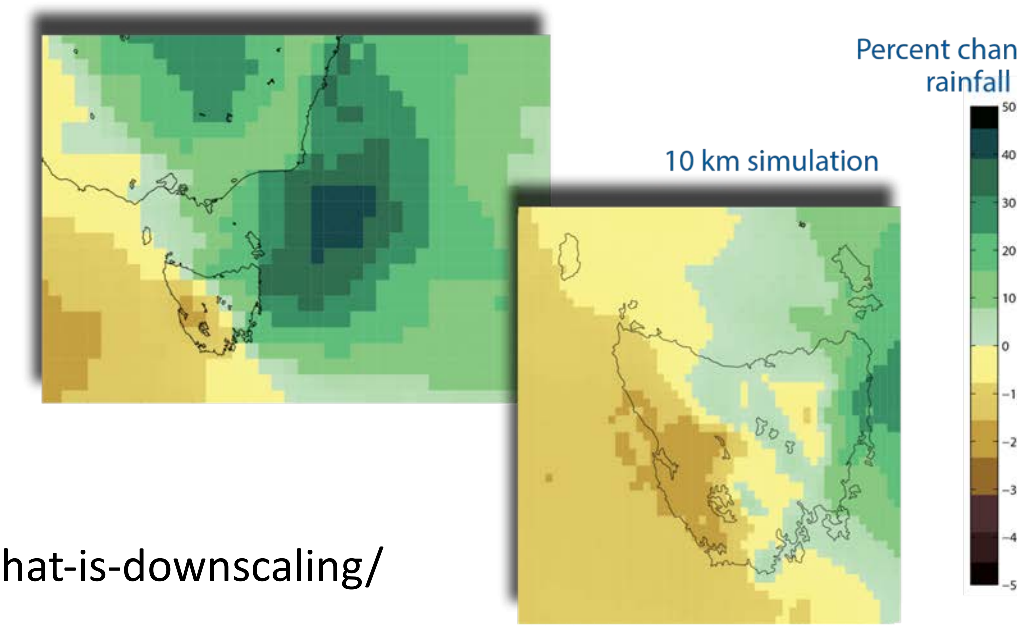



Neelin 2011

# Refined simulation/prediction/projection – with more local detail

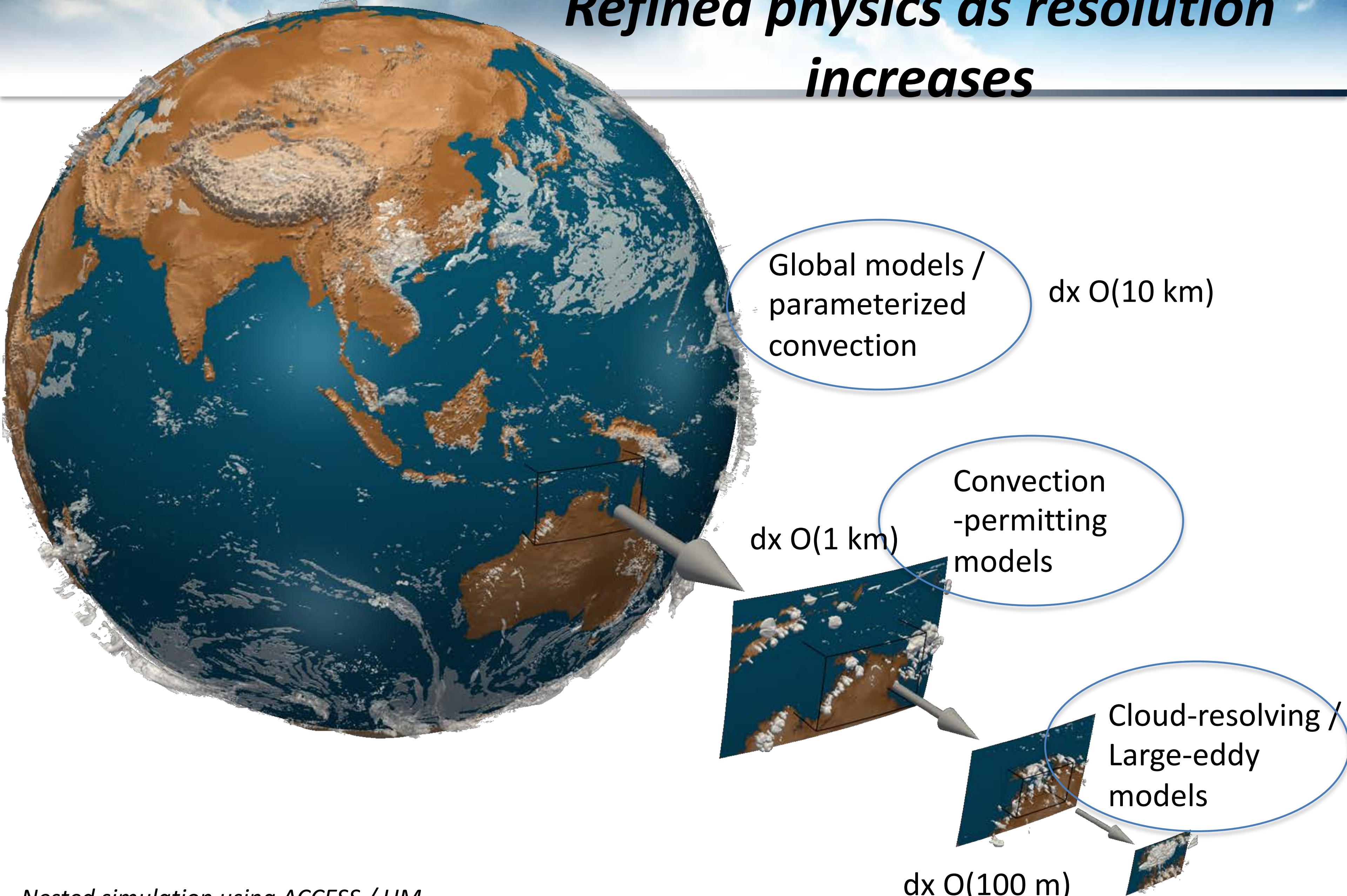
Global climate model





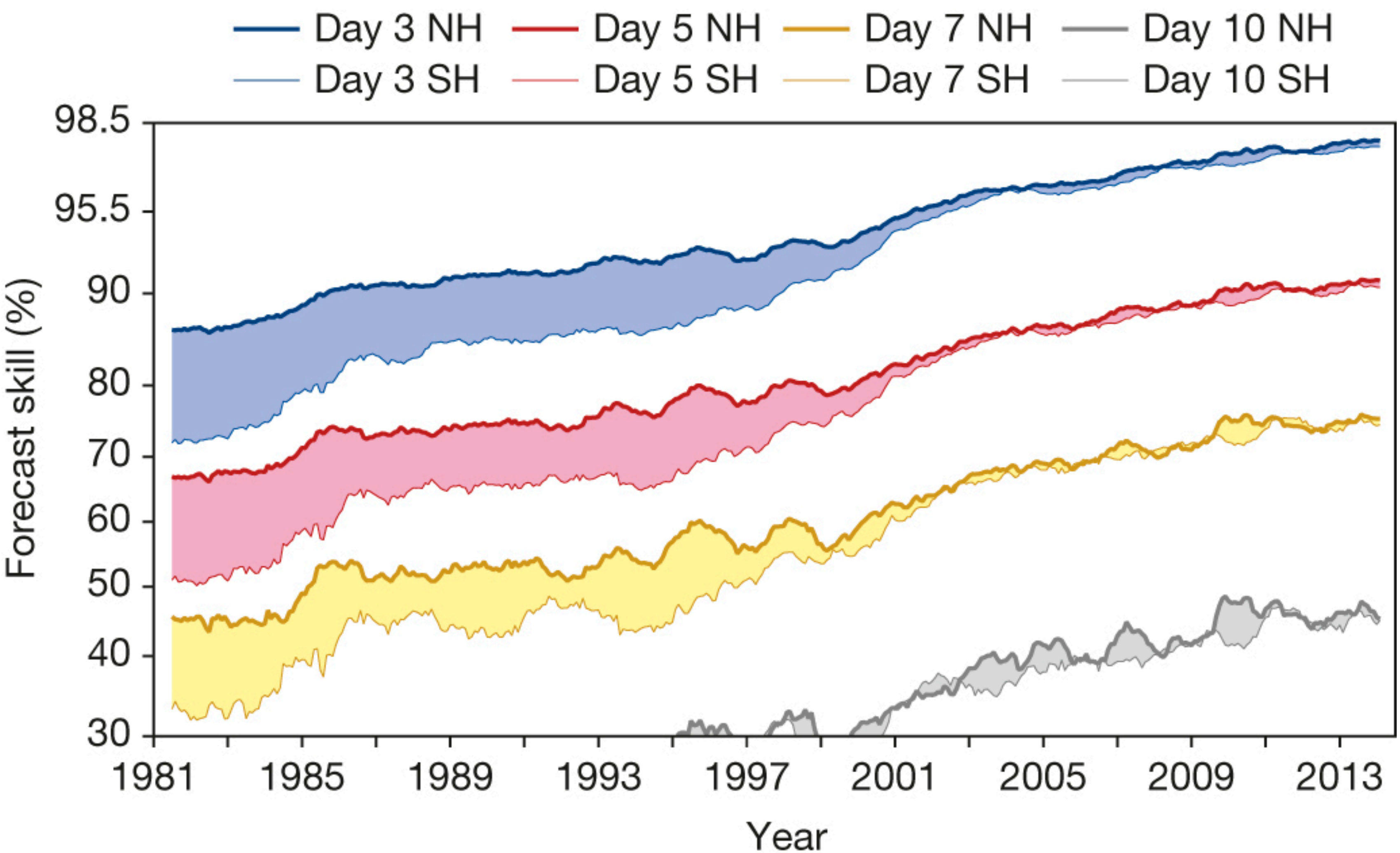

http://climatefutures.org.au/article/what-is-downscaling/

# *Refined physics as resolution increases*




*Nested simulation using ACCESS / UM*
*Courtesy of Martin Jucker*

# NWP model forecast improvement (model / data / assimilation)



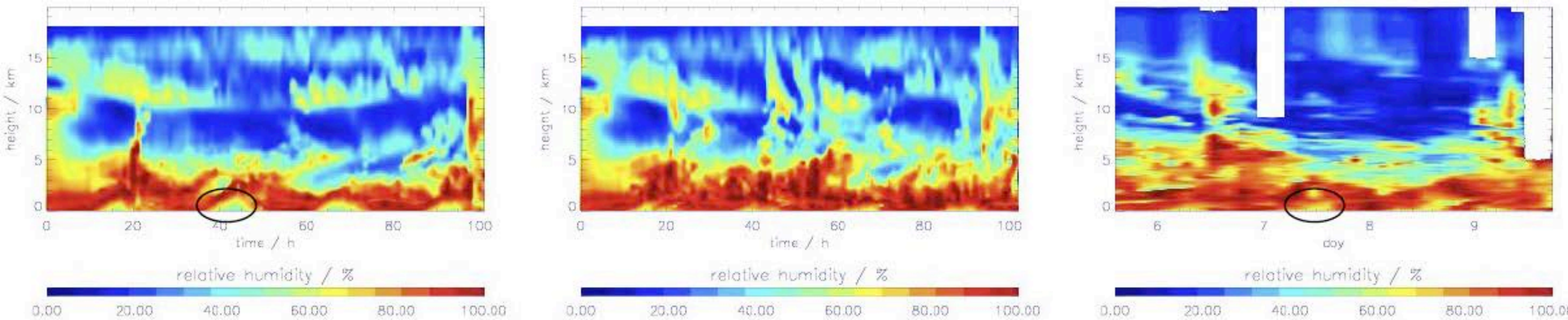

P Bauer *et al. Nature* **525**, 47-55 (2015) doi:10.1038/nature14956

# Sources of error

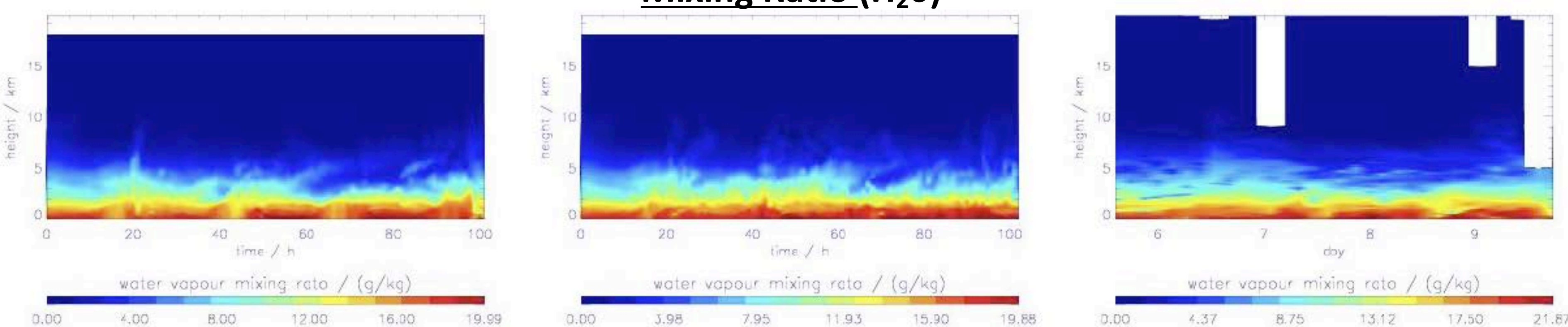
- Initial condition errors (for initial value problems [e.g., NWP / case studies], c.f. equilibrium experiments)
- Model physics (imperfect representation of the real world)
- Numerical errors (errors associated with numerical approximations and finite resolution –discussed later)
- Boundary condition errors (numerical as well as regional models constrained by imperfect large-scale conditions)

# Example: Errors in physics

**Relative Humidity**




**Mixing Ratio** ($H_20$)




WRF pbl=1

(YSU pbl scheme)

WRF pbl=2

(Mellor-Yamada-Janjic)

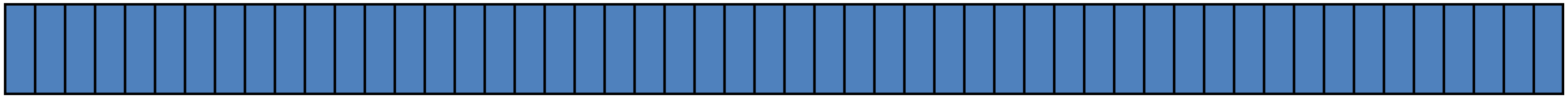
Point Stuart Radiosonde

(Wapler et al. 2010 simulations over Darwin with different boundary layer schemes)

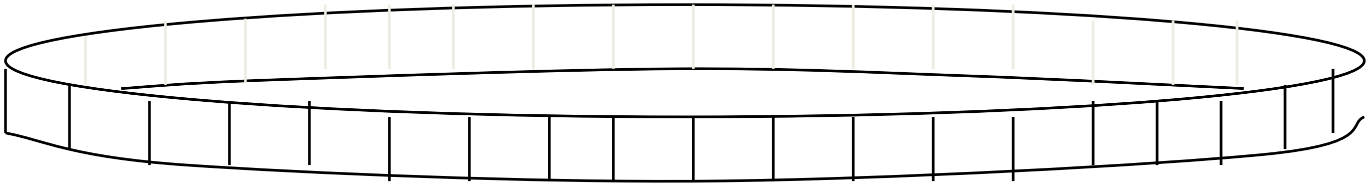
# Boundary Conditions..

**<u>Periodic boundaries (global model, idealized models)</u>**

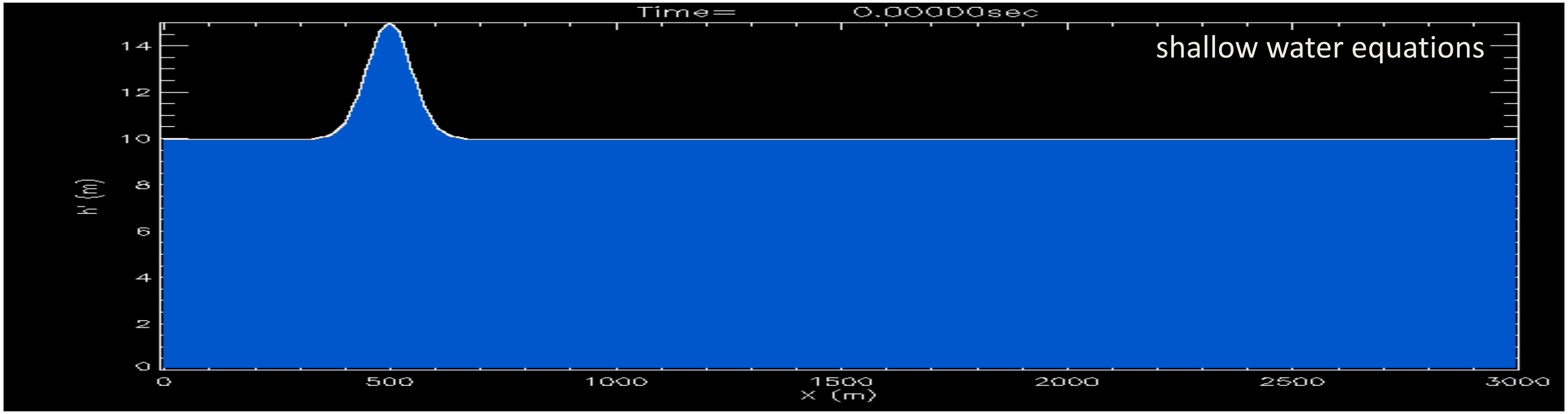
Assume that the domain is periodic or cyclic.



Periodic - repeats itself. - Equivalent to:



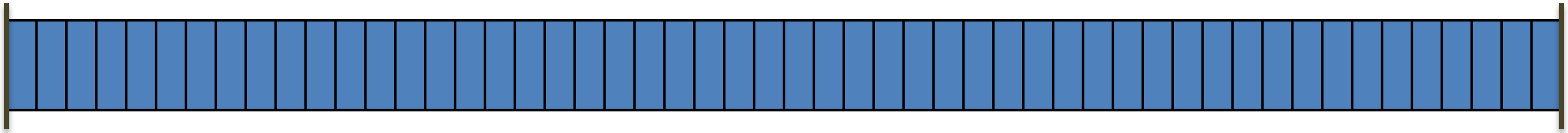
Periodic boundary conditions are <u>exact</u> (achieved through array indexing)



# Boundary Conditions..

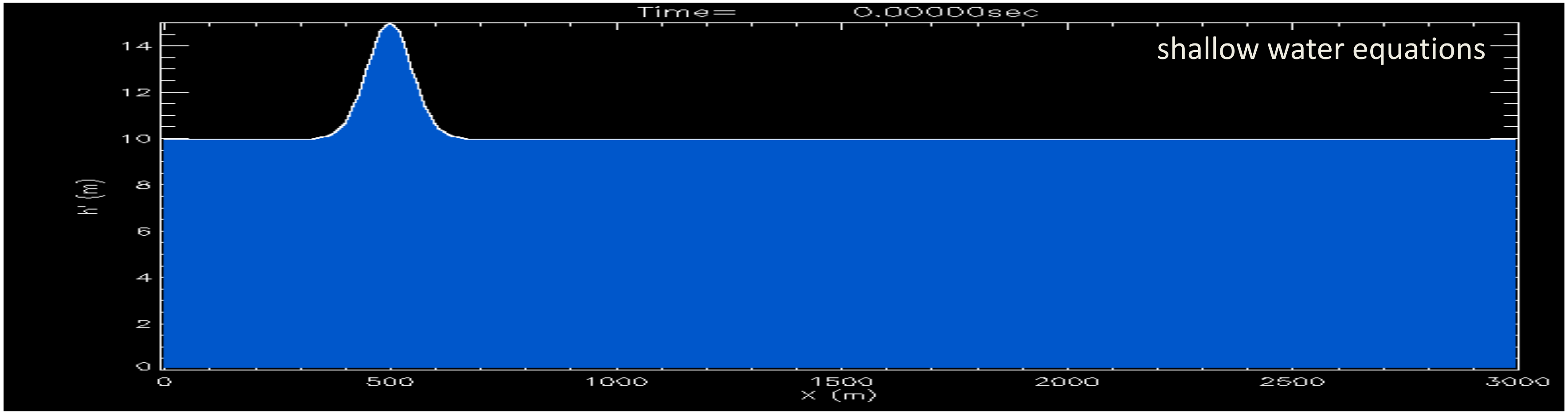
**Fixed boundaries (regional models, vertical boundaries, idealized models)**

Assume that the ends of domains are walls



Equivalent to u=0 at x=0, x=L, etc.:

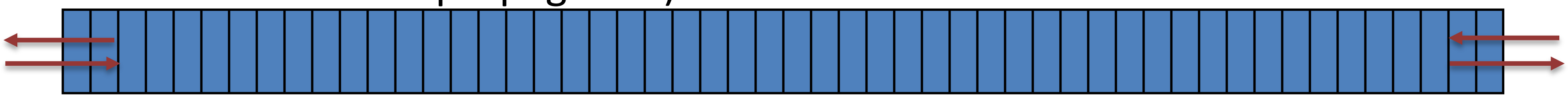
Fixed boundary conditions are exact (achieved through imposed velocities), but are perfectly reflective, which is not desirable for most atmospheric applications



# Boundary Conditions..

**Open boundaries (regional models, upper vertical boundary, idealized models)**

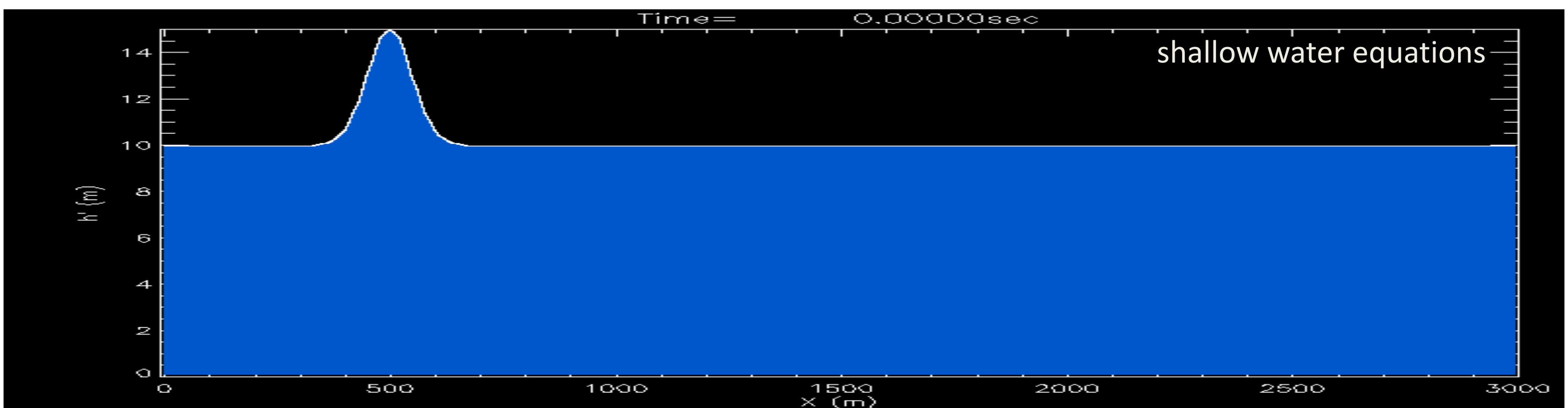
Assume that the ends of domains allow information in and out (both through advection and wave propagation)



Achieved through:

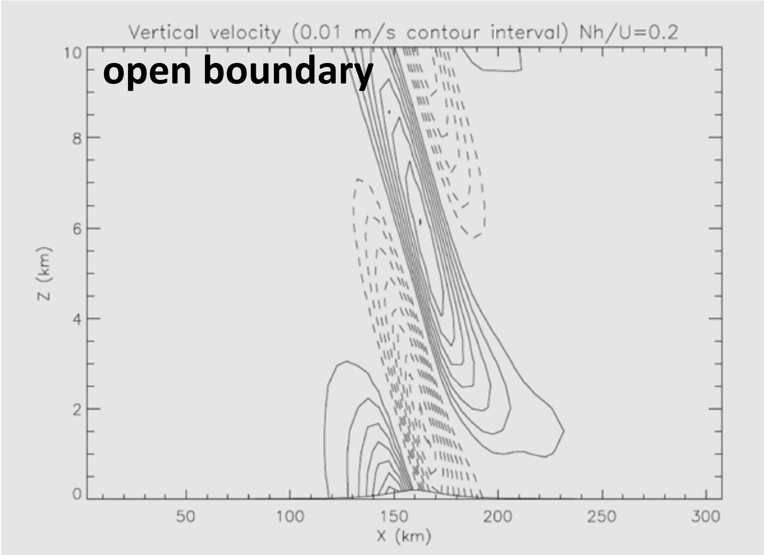
* solutions to wave equations for constant velocity / linear solutions
* sponge / absorbing layers; relaxation methods; extrapolation

Open boundary conditions are not exact in general and normally have spurious influences on the flow within some range of the boundaries and are partially reflective

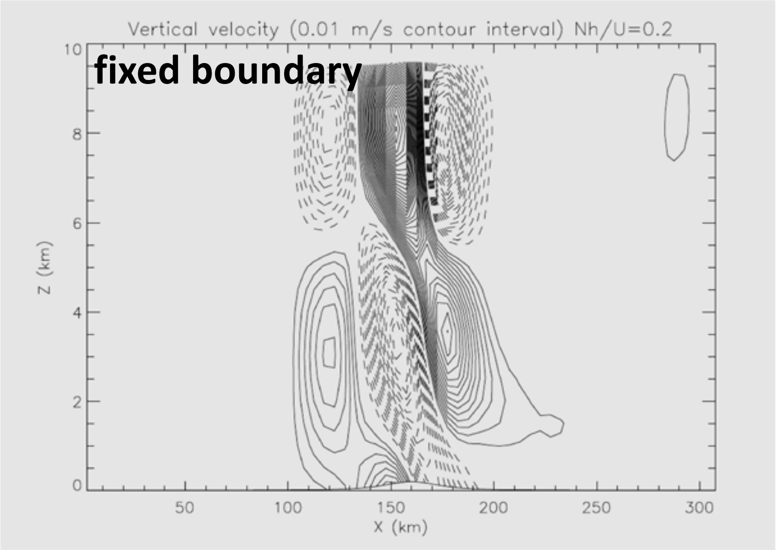


# Vertical Boundary Conditions..

- Lower boundary condition is an exact condition (fixed / reflective), where velocity normal to the surface is zero (e.g., w=0 on flat surface)
- Upper boundary condition should be open (to permit upward propagating waves to leave domain)
- Vertical grid is normally stretched as well – where grid spacing increases towards top of domain.
- Example: vertically-propagating mountain waves..




Reproduces analytic linear solution

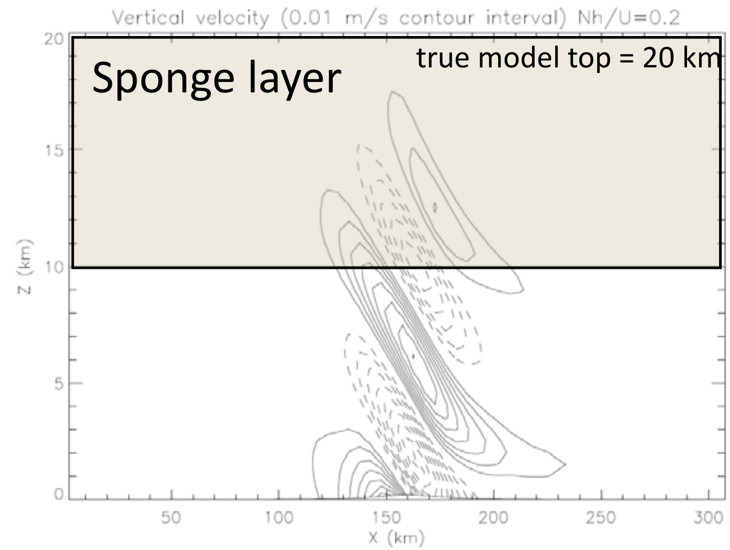



Large differences from analytic solution

# Vertical Boundary Conditions – sponge layer

- A layer that damps the perturbations in the flow before they reach the upper boundary.
- Can be used for vertical or lateral boundaries
- Removes boundary effects, and makes bottom or sides of sponge behave like an "open" boundary.
- Implemented as: $$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + B - \alpha w$$

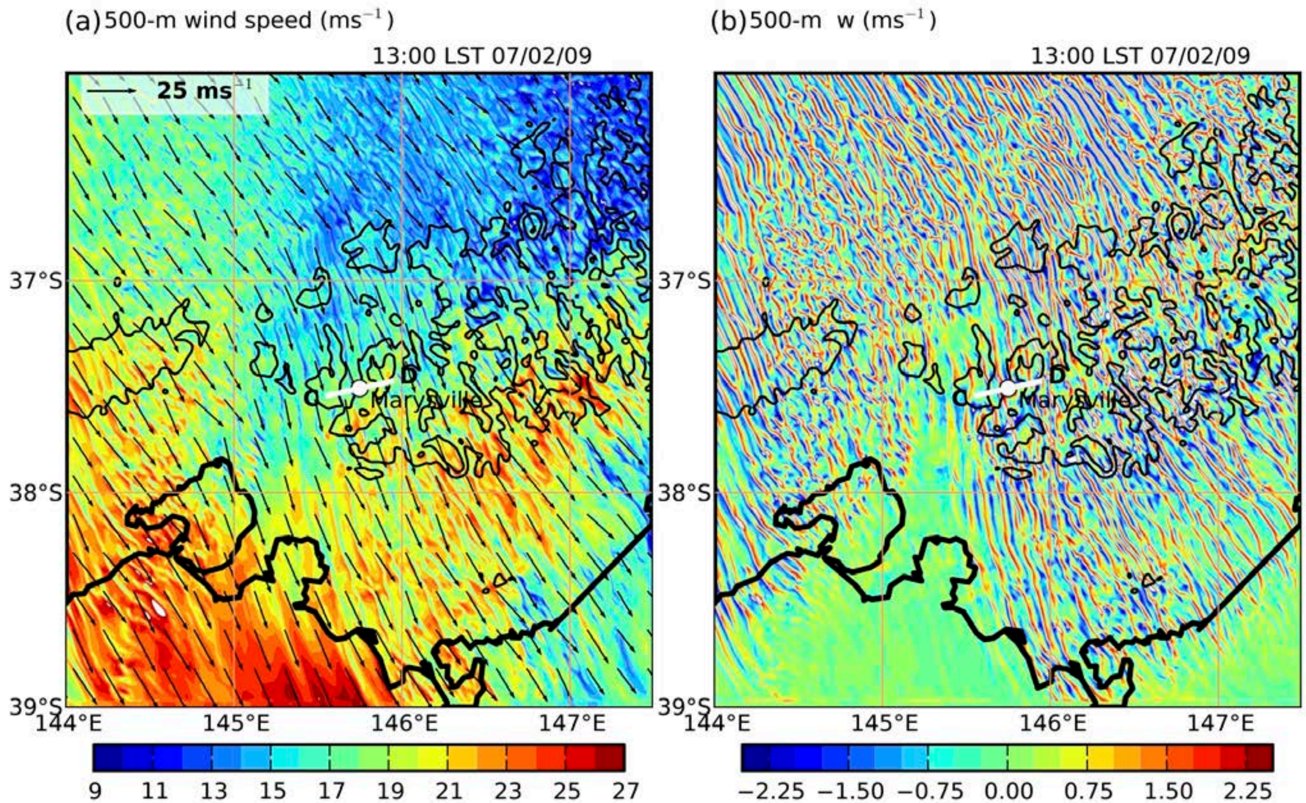
  $\alpha$ is damping - non-zero inside sponge layer and zero outside.
- Also called Rayleigh friction / Rayleigh damping
- Model solution inside the sponge layer is not a solution to the governing equations – should never be used!
- For this example half of entire domain is taken up by sponge.
  - Same amount of computer time is spent on sponge compared to usable solution, so very inefficient
- Can be used in combination with other open/radiation conditions to be more effective and efficient




*Most 'default' model upper boundaries in common community models and NWP models are partially reflective as the sponges are too shallow / too weak. This can pose problems for studies of upper-tropospheric, stratospheric, and wave dynamics*

## Lateral Boundary Conditions – regional model spin-up issues

- Regional (nested) models normally `feature' smoother features near their inflow boundary
- A property of larger-scale resolved flow coming from the coarse grid
- Distance of influence is related to advective timescale and timescale of growth of small-scale processes / instabilities
- Can alleviate this by seeding perturbations at boundary, but there is no unique way to do this




e.g., 50 km at 20 m/s takes about 40 mins, which is similar time for 5m/s thermal to span the 5 km deep mixed layer twice (eddy overturn time)

Engel et al. (2013)

# Fundamentals of Atmospheric Modelling

# Part 2
# (resolution and numerical issues)

**Todd Lane**

*ARC Centre of Excellence for Climate Extremes,*
*School of Earth Sciences, The University of Melbourne*

THE UNIVERSITY OF MELBOURNE

climate extremes
ARC centre of excellence

CLEX Winter School
24-28 June 2019

# Outline

- Parameterisation
- Effective resolution
- Implicit numerical diffusion and dispersion

# Parameterisation

## (parameterization [US], parametrization [UK])

Representation of unresolved processes using:

- Resolved scale flow.
- Some approximation, theoretical or empirical relationship linking the resolved flow to unresolved processes.

***"If this is the large scale flow, what is the small scale flow?"***

*or*

***"For a specific resolved flow, what is the unresolved flow, and how does this then feedback on the resolved flow?"***

- No unique ways to tackle this
- Approaches normally deterministic (but for many processes should be stochastic)
- Can be diagnostic or prognostic

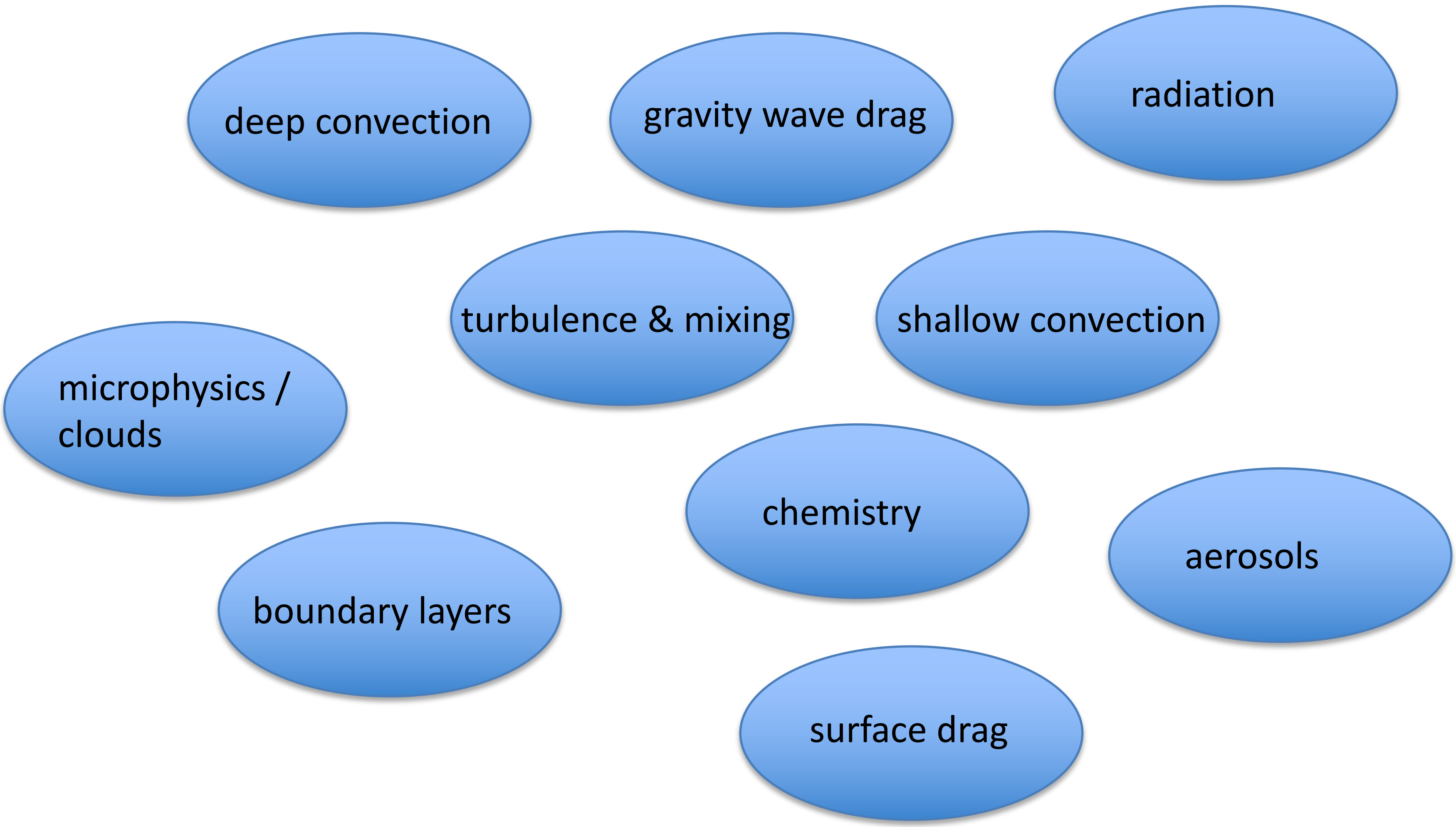
# Parameterisation

(parameterization [US], parametrization [UK])

*What needs to be parameterised?*

# Parameterisation

(parameterization [US], parametrization [UK])



# Why do we parameterise?

With finite grid spacing -

**Part of physical solution will be unresolved.**

e.g., 10 km grid spacing:

≥ 100 km signals will be well resolved (see next lecture)

< 20 km signals will be unresolved

20 -100 km signals will be partially resolved

.

Separate full signal, u, into resolved and unresolved part of signal.

$$u = \tilde{u} + u', \; w = \tilde{w} + w', \ldots$$

$\tilde{u}$ is resolved part of signal, $\langle \tilde{u} \rangle = \tilde{u}$ i.e., $\tilde{u}$ is a constant over a grid cell

$u'$ is unresolved part of signal, $\langle u' \rangle = 0$.

Similarly $\langle \tilde{u} u' \rangle = \tilde{u} \langle u' \rangle = 0$ $\quad\quad$ $\langle \; \rangle$ is average over grid

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Re – writing assuming $\underline{\nabla} \cdot \underline{u} = 0$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Recall $u = \tilde{u} + u' \quad w = \tilde{w} + w' \qquad p = \tilde{p} + p'$

$$\frac{\partial(\tilde{u} + u')}{\partial t} + \frac{\partial(\tilde{u} + u')^2}{\partial x} + \frac{\partial(\tilde{u} + u')(\tilde{w} + w')}{\partial z} = -\frac{1}{\rho}\frac{\partial(\tilde{p} + p')}{\partial x}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial u'}{\partial t} + \frac{\partial \tilde{u}\tilde{u}}{\partial x} + 2\frac{\partial \tilde{u}u'}{\partial x} + \frac{\partial u'u'}{\partial x} + \frac{\partial \tilde{u}\tilde{w}}{\partial z} + \frac{\partial \tilde{u}w'}{\partial z} + \frac{\partial u'\tilde{w}}{\partial z} + \frac{\partial u'w'}{\partial z} = -\frac{1}{\rho}\frac{\partial(\tilde{p} + p')}{\partial x}$$

Take average over a grid:

$$\frac{\partial\langle\tilde{u}\rangle}{\partial t} + \frac{\partial\langle u'\rangle}{\partial t} + \frac{\partial\langle\tilde{u}\tilde{u}\rangle}{\partial x} + 2\frac{\partial\langle\tilde{u}u'\rangle}{\partial x} + \frac{\partial\langle u'u'\rangle}{\partial x} + \frac{\partial\langle\tilde{u}\tilde{w}\rangle}{\partial z} + \frac{\partial\langle\tilde{u}w'\rangle}{\partial z} + \frac{\partial\langle u'\tilde{w}\rangle}{\partial z} + \frac{\partial\langle u'w'\rangle}{\partial z}$$

$$= -\frac{1}{\rho}\frac{\partial\langle\tilde{p}\rangle}{\partial x} - \frac{1}{\rho}\frac{\partial\langle p'\rangle}{\partial x}$$

$$\frac{\partial\langle\tilde{u}\rangle}{\partial t}+\frac{\partial\langle u'\rangle}{\partial t}+\frac{\partial\langle\tilde{u}\tilde{u}\rangle}{\partial x}+2\frac{\partial\langle\tilde{u}u'\rangle}{\partial x}+\frac{\partial\langle u'u'\rangle}{\partial x}+\frac{\partial\langle\tilde{u}\tilde{w}\rangle}{\partial z}+\frac{\partial\langle\tilde{u}w'\rangle}{\partial z}+\frac{\partial\langle u'\tilde{w}\rangle}{\partial z}+\frac{\partial\langle u'w'\rangle}{\partial z}$$

$$=-\frac{1}{\rho}\frac{\partial\langle\tilde{p}\rangle}{\partial x}-\frac{1}{\rho}\frac{\partial\langle p'\rangle}{\partial x}$$

$$\Rightarrow$$

$$\frac{\partial\tilde{u}}{\partial t}+\frac{\partial\tilde{u}\tilde{u}}{\partial x}+\frac{\partial\langle u'u'\rangle}{\partial x}+\frac{\partial\tilde{u}\tilde{w}}{\partial z}+\frac{\partial\langle u'w'\rangle}{\partial z}=-\frac{1}{\rho}\frac{\partial\tilde{p}}{\partial x}$$

assuming $\underline{\nabla}.\underline{\tilde{u}}=0$ gives

$$\frac{\partial\tilde{u}}{\partial t}+\tilde{u}\frac{\partial\tilde{u}}{\partial x}+\tilde{w}\frac{\partial\tilde{u}}{\partial z}+=-\frac{1}{\rho}\frac{\partial\tilde{p}}{\partial x}-\frac{\partial\langle u'u'\rangle}{\partial x}-\frac{\partial\langle u'w'\rangle}{\partial z}$$

Therefore, the **resolved scale motion is forced by the unresolved motion**. These terms on the RHS are called the subgrid-scale Reynolds stress terms.

**To be physically consistent, our model must incorporate some forcing from subgrid-scale processes – hence we need to parameterise them.**

# Parameterisation

**This Reynolds average approach is the basis of parameterizations of all dynamical processes (e.g., turbulence and mixing, boundary layers, surface drag, gravity wave drag, deep convection, shallow convection).**

**Other parameterizations of physical/chemical processes (e.g., microphysics, radiation, chemistry, aerosols) are slightly different as they are representing the grid-scale forcing/tendency from much smaller-scale reactions determined by the large-scale flow.**

**For example:**
radiation: $DT/Dt = F_{lw} + F_{sw}$ where the forcing is calculated by solving a set of integrodifferential equations with parameters determined by the resolved flow

microphysics: growth / decay of microphysical classes is determined by the grid-scale temperature and pressure, this leads to a tendency on the grid-scale water vapor mixing ratio and temperature

# Dynamical parameterisation example: Sub-grid scale mixing/turbulence

One approach: turbulence theory (K-theory) tells us that:

$$\frac{\partial \langle u'u' \rangle}{\partial x} \approx \frac{\partial}{\partial x}\left(-K\frac{\partial \tilde{u}}{\partial x}\right)$$

$$\frac{\partial \langle u'w' \rangle}{\partial z} \approx \frac{\partial}{\partial z}\left(-K\frac{\partial \tilde{u}}{\partial z}\right)$$

Provides a simple way to relate sub-grid terms to resolved part of the flow.

So our Reynolds averaged equation reduces to:

$$\frac{\partial \tilde{u}}{\partial t} + \tilde{u}\frac{\partial \tilde{u}}{\partial x} + \tilde{w}\frac{\partial \tilde{u}}{\partial z} + = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x} + \frac{\partial}{\partial x}\left(K\frac{\partial \tilde{u}}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial \tilde{u}}{\partial z}\right)$$

If K is a constant, this is the advection-diffusion equation.

K is called the sub-grid scale diffusion coefficient.

# Dynamical parameterisation example: Sub-grid scale mixing/turbulence

The system is still not closed as K needs to be defined. Defining K is the **'closure'**

For turbulence, K is defined by amplitude of expected turbulent motion.

K << 1 when flow is stable and laminar
K >> 1 when flow is turbulent.

Only way to determine K is using resolved scale features of the flow.

Many ways to do this.

e.g., Smagorinsky closure:
Ri is the Richardson number
Flow is turbulent if Ri<0.25

Smagorinsky closure

$$K = 0 \qquad Ri > 0.25$$

$$K = C\Delta x^2 |0.25 - Ri|^{1/2} \qquad Ri \leq 0.25$$

$$\text{where } Ri = \frac{N^2}{\left(d\tilde{u}/dz\right)^2}$$

# Dynamical parameterisation example: Sub-grid scale mixing/turbulence

**Types of ‘closures’**

- Zero-order closure:
  - assume <u’w’> =0, etc.

-1st-order closure:
  - parameterise fluxes as ‘diffusion processes’, as in previous example
  - diffusion coefficients determined from diagnostic relations

- 2nd-order closure:
  - derive prognostic equation such that:

$$\frac{D\overline{u'w'}}{Dt} = -\frac{\partial \overline{u'u'w'}}{\partial x} \ldots$$

  - the triple product terms would then need parameterising (in terms of double products). E.g.

$$\overline{u'u'w'} \approx -K\frac{\partial \overline{u'w'}}{\partial x} \ldots$$

  - this makes the scheme a prognostic scheme (as opposed to diagnostic)

# Convective parameterisation

APRIL 1974 AKIO ARAKAWA AND WAYNE HOWARD SCHUBERT 675

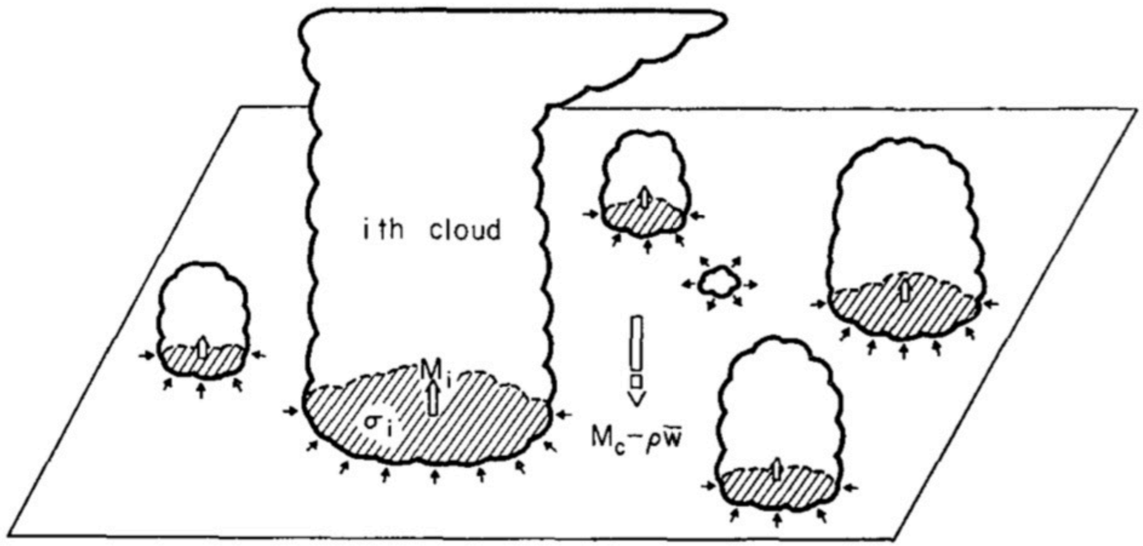
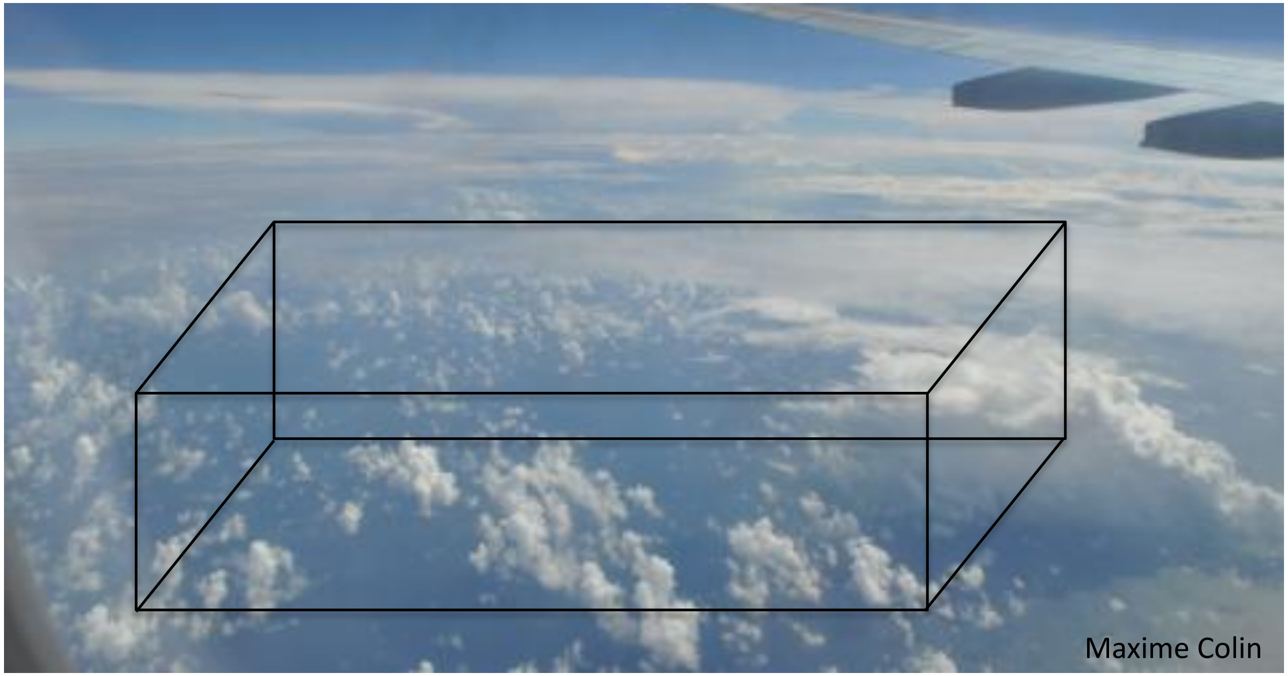



FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Parameterisation of the convective mass flux ($Mc = \rho\sigma w_c$, where $\sigma$ is the fractional area of clouds) for a population of clouds normally represented by idealized plumes

# Parameterisation assumptions

**For all parameterisations there is a fundamental assumption of a separation of scale between the resolved flow and the process being parameterised.**




- Parameterisation formulated on atmospheric columns (no ‘knowledge’ of state of adjacent grid boxes)
- E.g., convection: area of convective plumes much smaller than area of grid box
- For dynamical parameterisations equivalent to saying that many individual elements form part of Reynolds average

# The grey zone

As resolutions increase the parameterization assumptions are violated.

Scales where the parameterized processes become partially resolved

Large convective clouds - ~1-10 km

Boundary layer eddies and turbulence ~<1 km

Normally parameterisations are still used in ‘grey zone’ even though the fundamental assumptions behind those parameterisations are violated

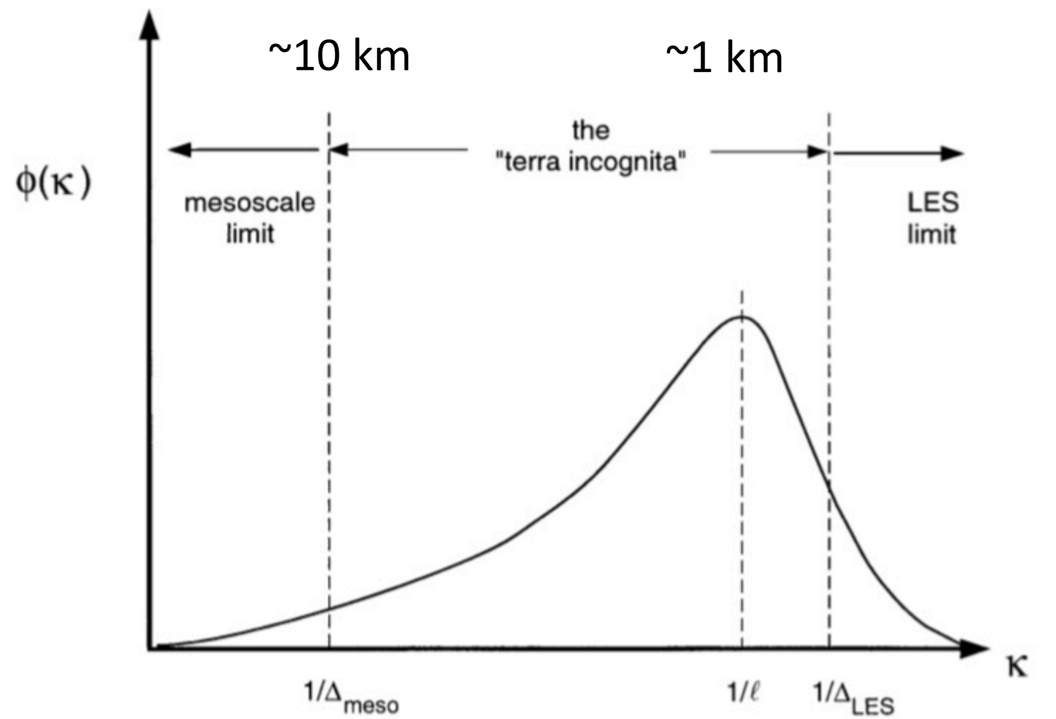



FIG. 1. A schematic of the turbulence spectrum $\phi(\kappa)$ in the horizontal plane as a function of the horizontal wavenumber magnitude $\kappa$. Its peak is at $\kappa \sim 1/l$, with $l$ the length scale of the energetic eddies; $\Delta$ is the scale of the smoothing filter. In the mesoscale limit (left), $\Delta_{meso} \gg l$ and none of the turbulence is resolved. In the LES limit (right), $\Delta_{LES} \ll l$ and the energy-containing turbulence is resolved.

Wingaard (2004, JAS)

# Effective model resolution

Models do not properly resolve the dynamics/physics they are trying to represent on the grid scale.

Numerical errors are maximized at the grid scale

Implicit and explicit numerical diffusion reduces the energy at the smallest represented scale

The effective resolution of a model ends up being about 7-10 times the grid scale.

- Resolution does not equal grid spacing!

**To properly resolve a phenomenon the grid spacing needs to (at least) be 10 times smaller than the scale of the phenomenon**

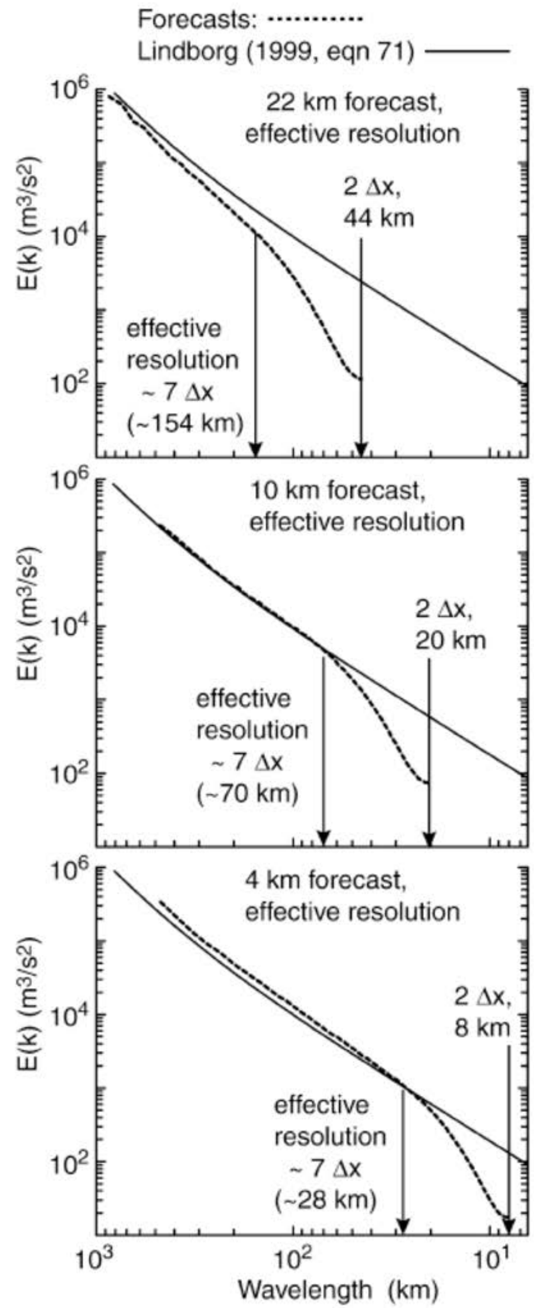



FIG. 11. Effective resolution determined from forecast-derived spectra for the BAMEX-configured WRF model at 22-, 10-, and 4-km horizontal grid spacing. The model forecast spectra are those plotted in Fig. 3.

Skamarock (2004)

# Example: cloud-resolving modelling

Squall line is ~ 20 km across

Inflow is ~ 5 km across

1 km model does not resolve the turbulent processes that should be formed by the convective instability

- Large eddy scale ~ 1 km

~100 m grid spacing model resolves the largest eddies (just) while smaller eddies are still parameterized

O(1 km) grid spacing models are now called 'convection-permitting' models as it is recognized that they don't properly resolve convection.

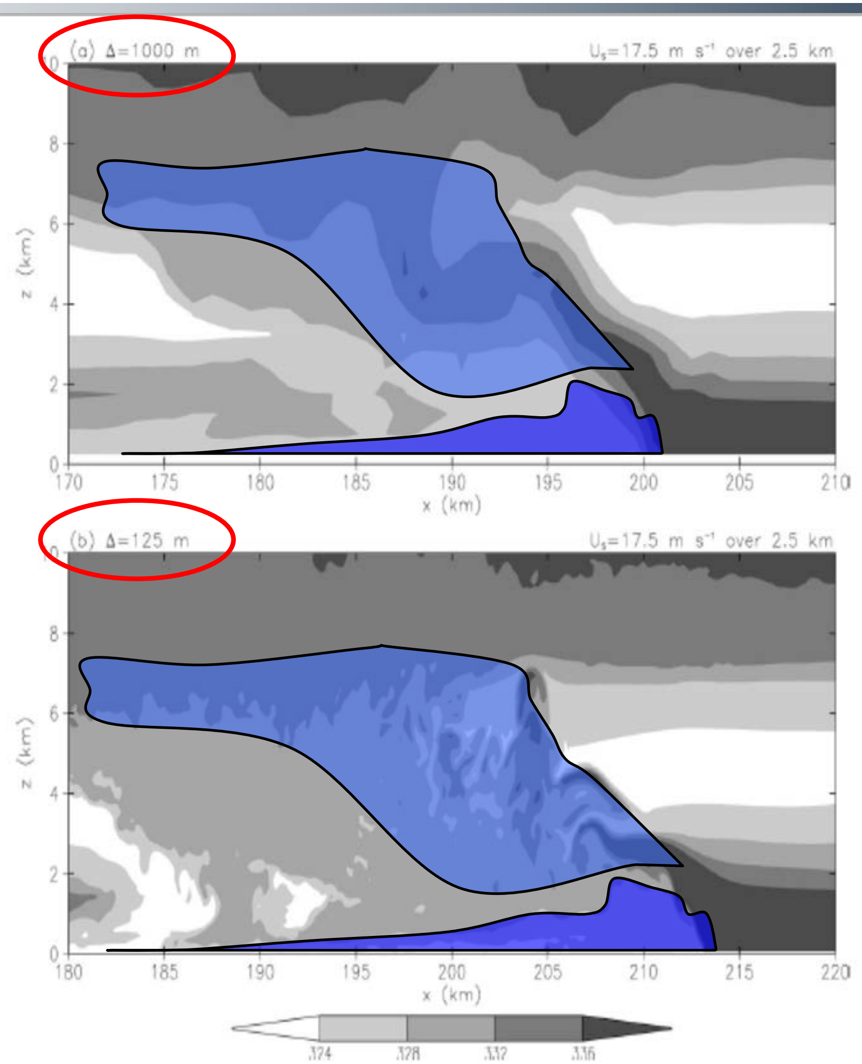



FIG. 1. Across-line cross sections of equivalent potential temperature ($\theta_e$, in K) from strong-shear simulations at 180 min using (a) 1000-m grid spacing (at y = 49) and (b) 125-m grid spacing (at y = 56 km).

(Bryan et al. 2003, MWR)

# Example: cloud-resolving modelling

Squall line is ~ 20 km across

Inflow is ~ 5 km across

1 km model does not resolve the turbulent processes that should be formed by the convective instability

- Large eddy scale ~ 1 km

~100 m grid spacing model resolves the largest eddies (just) while smaller eddies are still parameterized

O(1 km) grid spacing models are now called 'convection-permitting' models as it is recognized that they don't properly resolve convection.

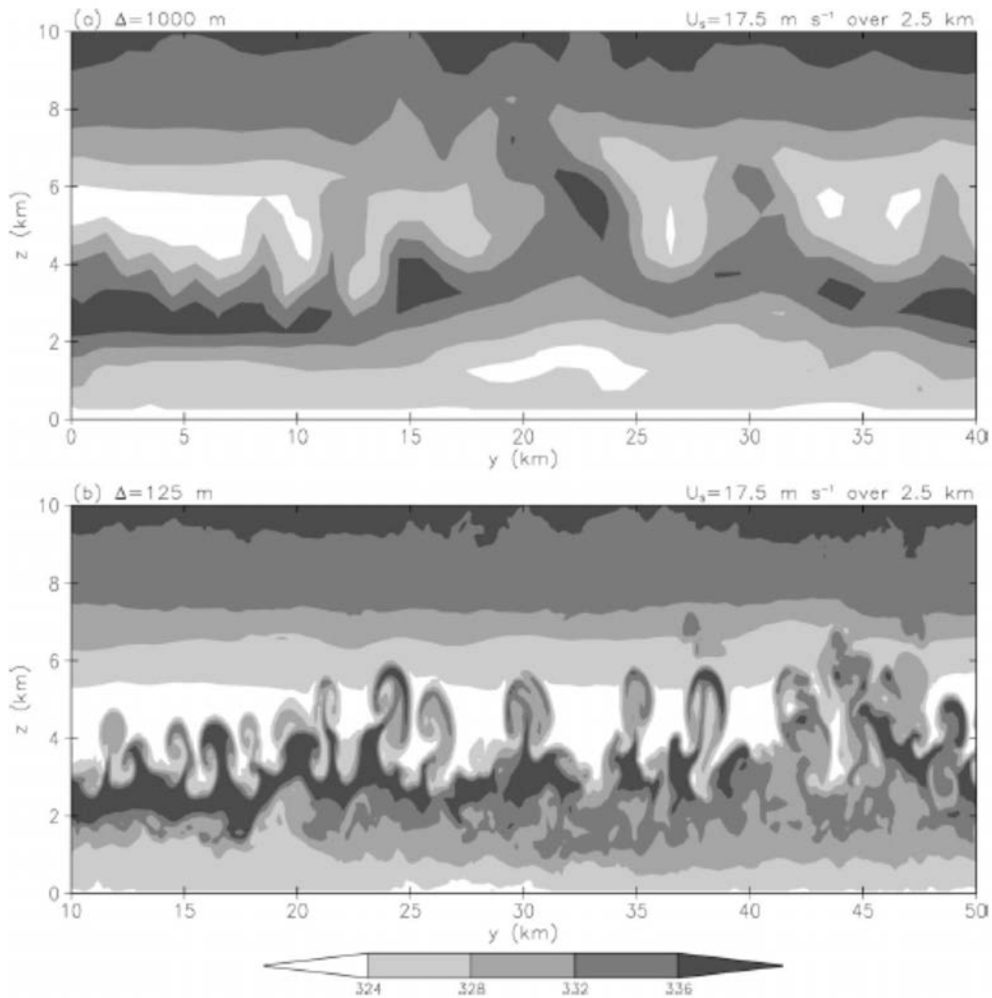



FIG. 2. The same as in Fig. 1 except along-line cross sections using (a) 1000-m grid spacing (at $x$ = 200 km) and (b) 125-m grid spacing (at $x$ = 207 km).

(Bryan et al. 2003, MWR)

# Understanding implicit numerical errors - a (very) brief introduction

Starting point for every numerical scheme is solving:

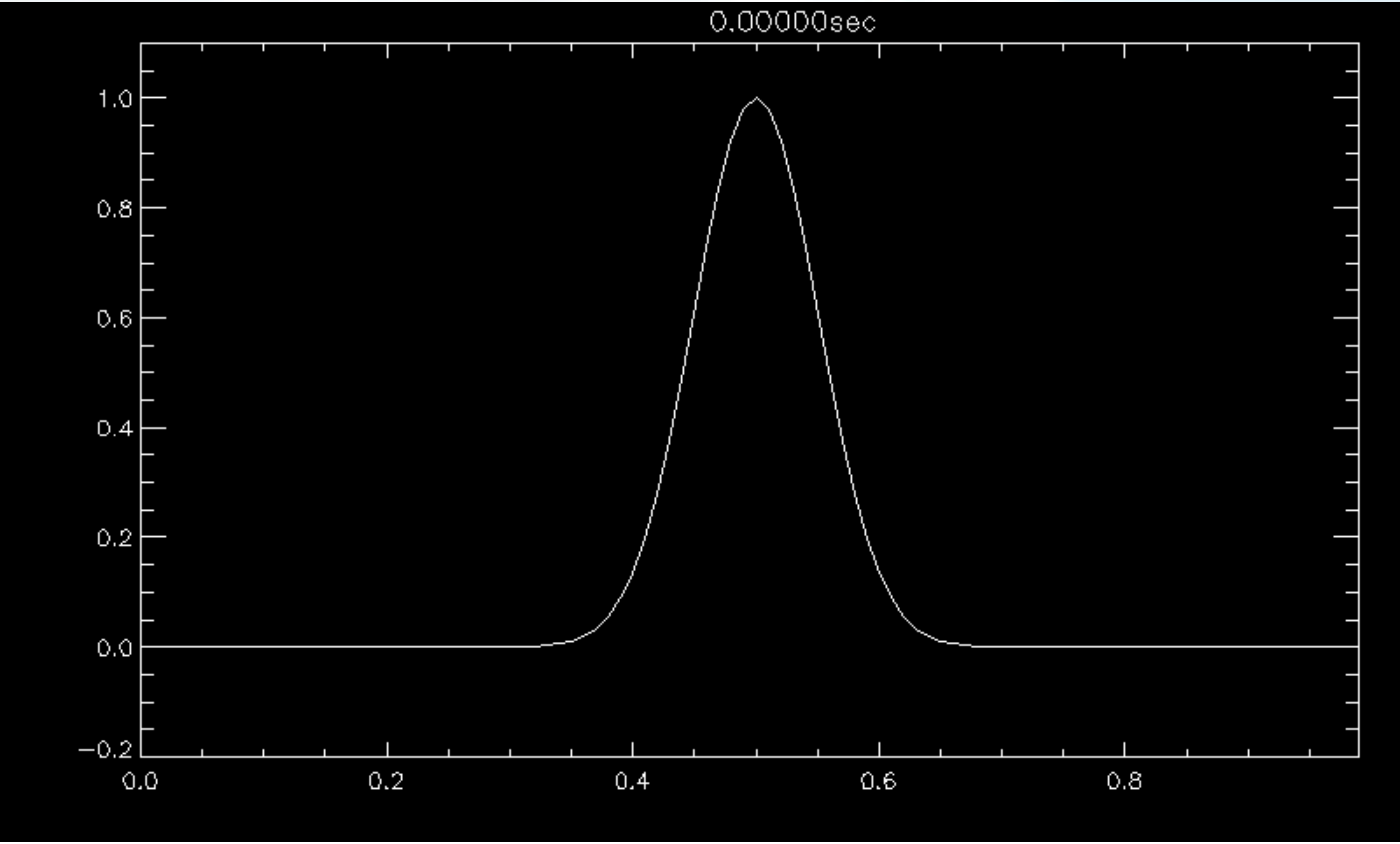
$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$

With *u* a known constant.

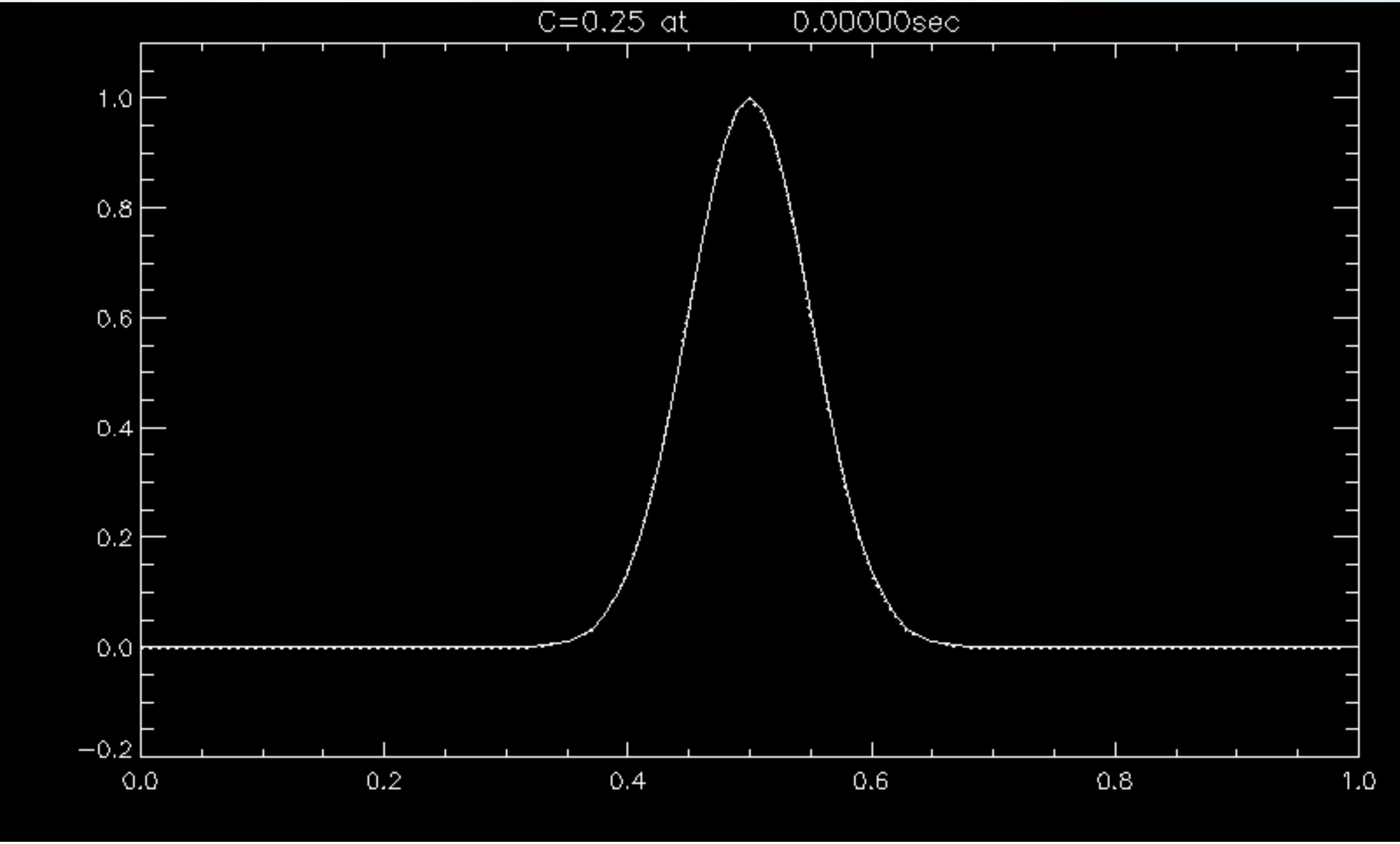
The choice of spatial differencing method strongly influences the solution.

*Example - advection of a bump*

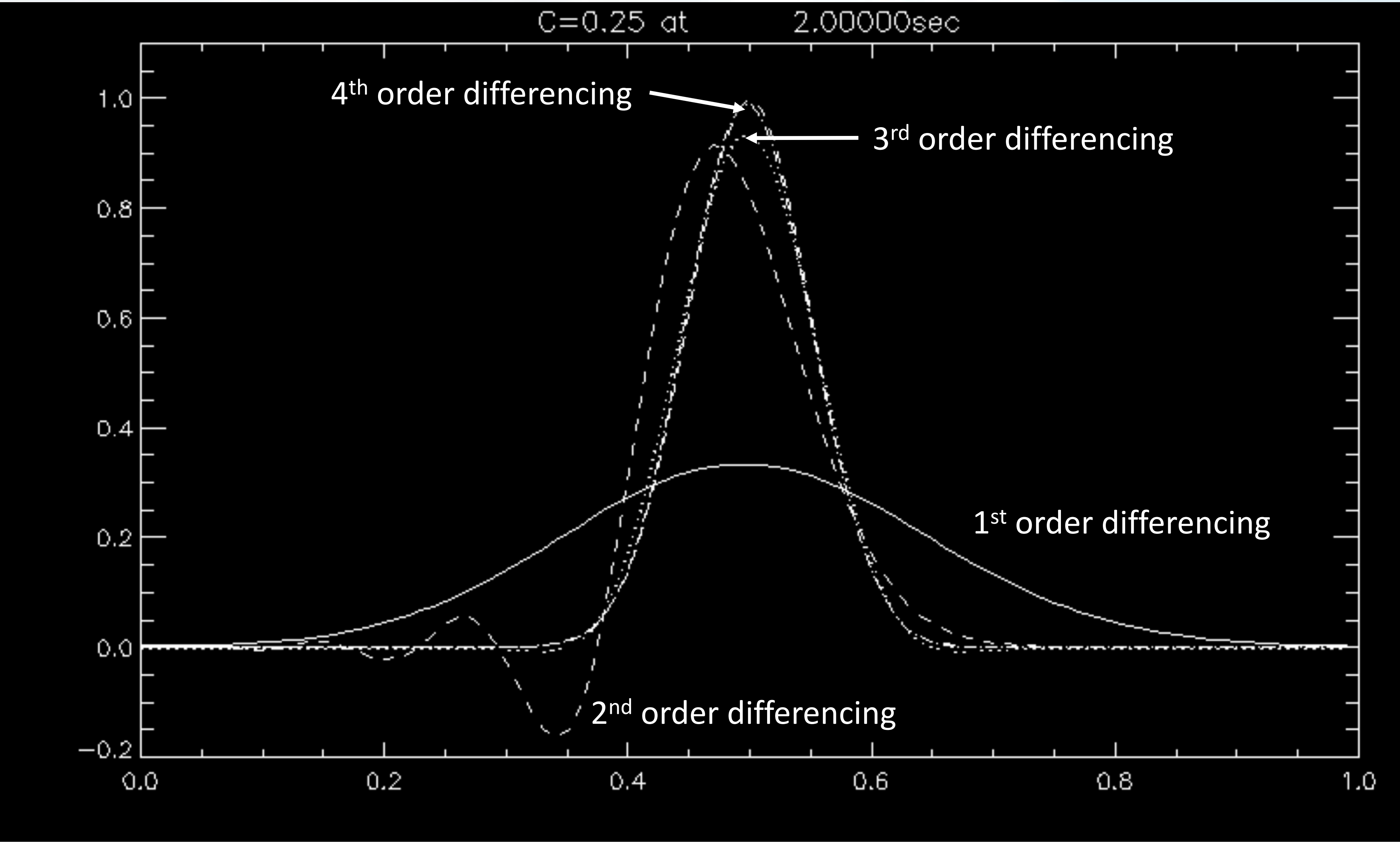
# Exact solution



# Numerical solution



# Numerical solution



## Summary of examples

Using high-order temporal differencing and different spatial differencing:

e.g., 1st order differencing.

$$\frac{\partial \phi}{\partial t} + u\frac{\phi_{i,n} - \phi_{i-1,n}}{\Delta x} = 0$$

**<u>Leading order error of differencing scheme controls solution</u>**

1st order method is strongly damping (diffusive)

2nd order method - signal travels too slowly & separates signal into component wavelengths (dispersive)

3rd order method - damping (slightly)

4th order method - travels too slow (slightly)

**General result that odd ordered schemes are diffusive and even ordered schemes are dispersive - truncation errors are important!!**

# 1-st order differencing

Using high order temporal differencing and different spatial differencing:

e.g., 1st order differencing $$\frac{\partial \phi}{\partial t} + u\frac{\phi_{i,n} - \phi_{i-1,n}}{\Delta x} = 0$$

This is our approximation to the 1-D advection equation. i.e.,

$$\frac{\partial \phi}{\partial t} + u\frac{\phi_i - \phi_{i-1}}{\Delta x} = \frac{\partial \phi}{\partial t} + u\left(\frac{\partial \phi}{\partial x} + O(\Delta x)\right) = 0$$

Remember

$$\frac{\phi_i - \phi_{i-1}}{\Delta x} = \frac{\partial \phi}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2)$$ therefore,

$$\frac{\partial \phi}{\partial t} + u\frac{\phi_i - \phi_{i-1}}{\Delta x} = \frac{\partial \phi}{\partial t} + u\left(\frac{\partial \phi}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2)\right) = 0$$

Retaining leading error term gives

$$\frac{\partial \phi}{\partial t} + u\left(\frac{\partial \phi}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 \phi}{\partial x^2} + O(\Delta x^2)\right) = 0$$

Therefore our 1st order approximation to the 1D advection equation is equivalent to

$$\frac{\partial\phi}{\partial t}+u\left(\frac{\partial\phi}{\partial x}-\frac{\Delta x}{2}\frac{\partial^2\phi}{\partial x^2}+O(\Delta x^2)\right)=0$$

$$\frac{\partial\phi}{\partial t}+u\frac{\partial\phi}{\partial x}=\frac{u\Delta x}{2}\frac{\partial^2\phi}{\partial x^2}+O(\Delta x^2)$$

The 1D advection-diffusion equation is

$$\frac{\partial\phi}{\partial t}+u\frac{\partial\phi}{\partial x}=K\frac{\partial^2\phi}{\partial x^2},$$ Where K is a diffusion coefficient

**Our difference equation is equivalent to solving the advection-diffusion equation with K=uΔx/2 - with second order accuracy.**

$$\frac{\partial \phi}{\partial t} + u\frac{\phi_i - \phi_{i-1}}{\Delta x} = 0$$

is a 1st order approximation to

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$ (the 1D advection equation*)

but a 2nd order approximation to

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \frac{u\Delta x}{2}\frac{\partial^2 \phi}{\partial x^2}$$ (the 1D advection-diffusion equation**)

**Our difference equation is a better approximation to the advection-diffusion (**) equation than it is to the advection equation (*)!**

For 2$^{nd}$ order scheme can show that leading order error term leads to dispersion.. i.e., different signals travel at different speeds.

The equation derived by including the leading order error terms (**) is called *the modified equation*. The modified equation determines the actual form of the solution!!

## Importance of understanding behavior of numerical methods

- Truncation errors have a large (dominant) effect on the solution at the smallest resolvable scale

- Whether a scheme is diffusive or dispersive matters
  - Dispersive solutions can be noisy – which can be important for one-signed variables (e.g., water vapor mixing ratio)

- Additional diffusion is normally imposed to suppress grid-scale noise

- Care must be taken interpreting any model output at scales less than ~10 Δ as the spatial / temporal variability at these scales is controlled by numerics

# Summary

- Atmospheric models are complicated
- Often treated as ‘black boxes’ but understanding their construction, assumptions and limitations is important
- All aspects of atmospheric models are imperfect, but some parts are less perfect than others – this depends especially on the scales of motion you are considering

# Summary